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ABSTRACT

The concern of this publication is to explore the extent to which there is common ground in the approach to the education of primary school teachers in mathematics and science. Chapter 1, "A Model for Discussing Teacher Education" outlines a rationale for making decisions about approaches to teacher education. Chapter 2, "The Interface Where Science and Mathematics Meet and Mingle" presents two examples of how mathematics and science concepts can be taught simultaneously. Chapter 3, "Children's Learning in Science and Mathematics" briefly considers characteristics of learning in mathematics and science. Chapter 4, "What Should Teachers of Primary Mathematics and of Primary Science Know?" explores how teachers should develop an intellectual structure of interrelated concepts on which they can draw with confidence when placed in a problem situation. Chapter 5, "Education of Assessment as Part of Teaching" provides a brief discussion of features and various purposes of assessment. Chapter 6, "The Training Needs of Teachers" suggests the full range of skills and abilities which primary teachers need. Chapter 7, "Teacher Education Approaches" presents ways in which teacher education programs can develop workshops that exploit the commonalties that exist between mathematics and science while providing differences where appropriate. The production of these chapters was a combined effort of the following authors and contributors: Jos Elstgeest, Fred Goffree, Wynne Harlen, Hubert Dyasi, Bernard Hodgson, and Jeanne Bolon. Contains 65 references. (ZWH)

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edited by Wynne Harlen

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Introduction

The concern of this small publication is to explore the extent to which there is common ground in the approach to the education of primary school teachers in science and mathematics. To the extent that this is the case it may be possible for there to be some economy in time and effort and some increase in effectiveness in teacher education programmes. Given the greater emphasis on these areas of the primary curriculum in all countries, and the concern that this causes to teacher education, such possibilities are well worth serious exploration. But the arguments go beyond the pragmatic to the essential nature of the subjects and the value of defining differences and commonalities so that the identity of each subject is respected even in the early stages of children's learning. We have been careful to respect the integrity of the mathematics and science whilst arguing that there is much in common about the approach to learning the subjects at the primary level.

Two things are best made clear at the start. First, that it is not our concern to advocate an integrated approach to science and mathematics in school teaching. Second, that we are neither advocating nor not advocating an integrated approach in teacher education; our purpose is to raise and explore relevant issues. We would not have embarked on this study, however, if there were not reasons to consider that there exist major areas of similarity and real possibilities of improvement in teacher education that may result from considering the similarities and differences between science and mathematics in teaching and learning at both school pupil and teacher education level.

A major area of similarity is the view of learning of the subjects and the way in which this is conveyed in teacher education. It is recognised that a view of learning is communicated to teachers or student teachers not only overtly, through providing information, but as a message implicit in the way in which courses are conducted. The particular view of learning which is shared in mathematics and science education and by the authors emphasises the importance of taking the learner's initial ideas as a starting point and the participation of the learner in modifying and extending ideas in the light of experience. If this constructivist view of learning, perhaps as one of others, is to be embedded in the training of teachers, there are significant consequences for training programmes. Of similar importance is the way in which the nature of the subjects is conceived. These factors seem to the present authors to be of such fundamental importance that we have used them as the basis for the structure of this publication.

We have begun, therefore, in Chapter 1, by presenting a model and a rationale for the significant influences on teaching. What is significant in teaching necessarily identifies important foci for teacher education.

Consistent with the important role we have ascribed to the view of the subject and of the nature of learning in science and mathematics, the next two chapters are concerned with these matters. Chapter 4 addresses the vexed question of the knowledge of science and mathematics that primary teachers need. The emphasis here is on an adequate basic grasp of different aspects of the subjects rather than on advanced mastery of an academic kind.

Assessment is a third major influence on teaching. This is addressed in Chapter 5, where emphasis is laid on assessment as a formative part of teaching. The implementation of the constructivist view of learning, which underpins the thinking in this publication, depends on assessment of pupils' ideas and skills. Chapters 6 and 7 discuss more directly the content and methods of teacher education programmes. Chapter 6 offers lists of the opportunities for professional development which might be provided at pre-service and in-service stages. What is learned, however, depends not just on the content of courses but on the methods used in training, matters which are taken up in more detail in Chapter 7.

To reiterate an earlier statement, these chapters do not offer solutions or lines of action to be followed. Their purpose is to provoke thinking and discussion of issues relating to the education of future teachers of primary science and mathematics. In writing them, we ourselves found that we raised many questions which we were not able to address and identified areas where further research and development are needed. We hope that by listing these at the end of the booklet the publication may more easily be used as a discussion and study document by others. To further this purpose, we also include an annotated bibliography of sources which others may find as useful as we have. Whilst there is a degree of logic to the order of the chapters, sequential reading is not essential. To aid 'dipping', each chapter begins with a brief summary of its contents.

The production of these chapters has been a combined effort of the contributors, listed on page 2. The publication began as an idea discussed among representatives of UNESCO, ICMI and ICSU-CTS in March 1989. Writing began in earnest after a workshop held in Liverpool in February 1990 and subsequent drafts were further discussed and refined at a meeting in Edinburgh in October 1991. Certain members of the group have taken the main responsibility for writing various chapters and are identified by their initials at the end of each chapter. During the whole processs and particularly at the final meeting, comments from others helped to shape the contents of the chapter and often to modify and add to them. In addition, a firm editorial hand had been applied to try to produce a coherent and readable whole.

Wynne Harlen

January 1992

1 A Model for Discussing Teacher Education

Summary: The purpose of this first chapter is to outline a rationale for decisions about approaches to teacher education. We begin with a model of teachers' decisions in the classroom and what may influence them. This model is used to indicate the aims of teacher education in preparing teachers for taking decisions about promoting learning in children. It suggests that what we prepare teachers to do depends on which kind of learning we want them to bring about, and the issue of values cannot be avoided. However, our point here is not to prescribe what should be done but rather to describe a model for decisions making.

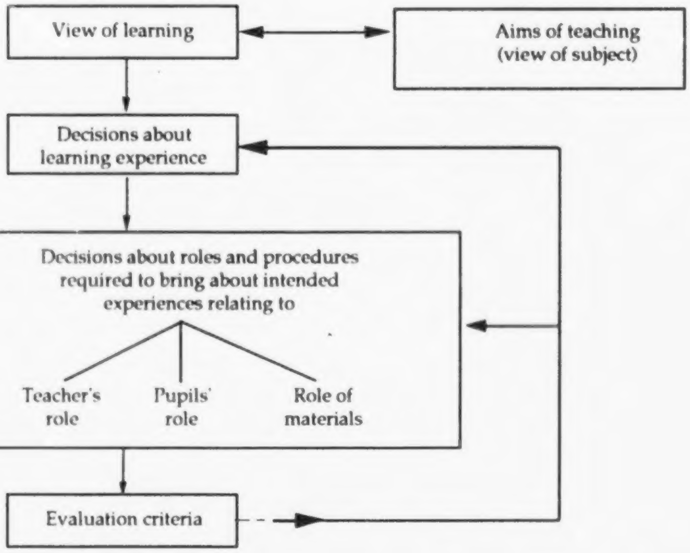
A model for classroom decision making

Observations of teachers in their daily work in classrooms indicate a consistency between how teachers go about their work and their views of learning. This has been recognised through research and is described as teaching style or approach[1,2]. One teacher will prefer to provide a range of activities for children, perhaps covering the whole curriculum, and will give the children a considerable amount of responsibility for choosing and completing their work. Another will keep the class as a whole for most of the time so that they will share the maximum amount of the teacher's attention. Yet another may encourage groups to work together and expect a cooperative product in some appropriate form. When asked about why they choose one approach rather than another, the reasons teachers give are in terms of their views of what they want children to learn and what they think is the best way of helping children to learn it. Each conscious decision about how to arrange the class, what kinds of activities to provide, how to bring children into interaction with the materials supplied, the kind of help the teacher gives and how success is to be assessed, will be consistent with the teacher's view of what and how children should be learning.

For example, suppose that a particular teacher's view of learning is that it is a matter of rote memorisation. This teacher will provide learning experiences which expose children to accurate facts and encourage them to memorise procedures and algorithms. To do this efficiently the teacher will probably provide the information in digestible packets, each to be mastered before the next is attempted. The class will be arranged to optimise exposure to information from the teacher, from the blackboard and from books, and to minimise interference from non-authoritative sources, such as other children. The teacher's role will be seen as being to ensure attention, to present information clearly and to reward accurate recall; the pupils' role is to attend, to memorise and to recall; resources may be used to illustrate applications of facts already learned or just to add interest and prevent boredom. Of course, the evaluation criteria will be defined in terms of how well the children can recall or reproduce information. This teacher will be doing a good job of rote teaching; all the classroom conditions are consistent with rote learning and support its implementation.

If the teacher has a different view of learning, along the lines suggested in the Introduction and described in more detail below, where the learner is active in creating understanding and in testing and modifying initial ideas, then the classroom provision consistent with it will clearly be quite different from that described for rote learning. Now the experiences provided will enable pupils actively to seek evidence through their own senses, to test their ideas, to take account of others' ideas through discussion and using sources of information; the class organisation will facilitate interaction of pupils with materials and pupils with pupils; the teacher's role will be to help children to express and test their ideas, to help them to reflect upon evidence; the pupils' role includes some responsibility for learning and taking part in generating ideas; the materials have a central role in providing evidence as well as arousing curiosity in the world around. The assessment criteria must include reference to developing and using skills and ideas and not neglect the development of related attitudes.

The model below (adapted from Harlen and Osborne, 1985)[3] is an attempt to convey this relationship between the kinds of learning experiences teachers try to provide for their pupils and the kind of learning they want to bring about. Their decisions about implementing this, in terms of the role they take as teachers, the role they allow for the pupils and the way in which they use resources, are made so as to be as consistent as possible with the intended learning experiences. The success of the teaching and learning is judged against criteria which in turn are related to the view of what should be learned and how it should be learned. Feedback from evaluation brings the planned experiences and their implementation more closely into line with the kind of learning intended.


View of learning
Aims of teaching (view of subject)
Decisions about learning experience
Decisions about roles and procedures required to bring about intended experiences relating to
Teacher's role
Pupils' role
Role of materials
Evaluation criteria

The classroom of the teacher who values rote learning and that of the one who takes a constructivist view would clearly be very different learning environments. Assuming that we do not favour rote learning, a question for in-service teacher education is: how can we turn the first one, arranged for rote learning, into a different one, perhaps more like the second? At this point we must admit that we have ignored the inevitable constraints on teachers imposed by conditions such as class size and the provision of materials and basic facilities, which are not available to all teachers. Where teachers lack the conditions which would enable them to teach in the way they would like, the solution has to be sought through other channels than teacher education.

However, if the teacher is free to choose and has chosen to teach by rote, the answer to our question about how to change the learning environment (s)he has created is certainly not by telling him or her to reorganise the class so that, for example, the children sit in groups. Nor will it be by merely providing different books, since a devotee of rote learning can turn anything into an exercise of memorisation. What is required is for the teacher to become re-educated as to the nature of learning. Only when the teacher is convinced that rote learning will not lead to understanding and is equally convinced of the value of children taking an active part in their learning will he or she be prepared to make the effort to organise the class, the materials, and to make the time, for children to learn in a way consistent with this view.

Teacher education: determining the view of learning and teaching

The above model proposes that the views of learning and of the subject are the keys to the learning opportunities that a teacher provides for the children. It therefore follows that these views must be given a high profile in teacher education. The major question in the present context is: to what extent are the views of learning relevant to primary school mathematics and science the same?

Let us consider the constructivist view of learning as an example, first taking its application in science and, later, in mathematics education. This is a view of learning which holds that the learner, in trying to make sense of new events or objects, begins from relevant existing ideas or models and tests the extent to which the new phenomena can be explained using these existing ideas or models. If predictions based on a related existing idea or model fits the new observations, then the range of application of the idea or model is extended; if the evidence does not fit the prediction, however, this may mean that the idea or model has to be modified or rejected in the light of the new evidence.

Science encompasses the first-hand use of physical and mental skills to generate and test reliable knowledge and generalisations. In learning science, these skills (referred to as the process skills) are involved in using and testing existing ideas. It is through processes such as observation, questions raising and hypothesising that ideas are used in trying to explain new evidence; it is through processes such as prediction, planning, experimenting and interpreting that conclusions are drawn as to whether the ideas fit the evidence. If these process skills are not carried out in a rigorous and scientific manner, then the emerging ideas will not necessarily fit the evidence. Ideas may be accepted which ought to have been rejected, and vice versa. Thus, the development of ideas depends crucially on the processes used. While the facts or data and generalisations are important, how we come about them and what makes us believe them are of equal importance. It is important that they derive directly from the phenomena themselves, that careful planning, observation and recording are done, and that the conclusions drawn are bases for further investigation and verification.

It follows that attention needs to be given to the way in which children test out their ideas in primary science, that is, to the development of the process skills. However, this cannot be done effectively by direct teaching any more than the understanding of abstract scientific concepts can be taught directly. Experience of attempting to give instruction in observation, or prediction, as such, using content-free activities (meaning trivial content, since there has to be some) is that the skills are not transferred to use in scientific enquiries as hoped. The usefulness of the skills in helping understanding has to be experienced. So a pupil who recognises that finding a pattern in observations has helped in making a useful prediction is likely to try this in another situation because of its value in helping understanding and not just because (s)he knows how to do it. The continual interweaving of knowledge and process skills in the investigation of natural phenomena is an essential characteristic of science education.

Science also involves using the knowledge that has been generated through process skills to create and continually refine testable models of nature that help us to describe, explain, predict, and to conceptualise observable phenomena of nature. In this model building of science, the approach is first-hand enquiry built around experience and experimentation and the focus is the phenomena themselves. The models at first will be approximations that are improved or revised or discarded in the light of additional data that comes available as children's experience expands. Children, like scientists, must be ready to reject ideas when the evidence requires this. In this way ideas gradually change and develop to be more encompassing, more generalised and more abstract.

In the type of learning just described the learner collects the evidence and does the reasoning; makes the ideas his or her own. This is what we may call learning with understanding. Learning without understanding, as in rote memorisation, does not require the use of process skills. Learning with understanding helps children to feel at ease with science, to know its strengths and weaknesses, to realise how ideas emerge from human activity, which is important in their education even if they are not destined to practise science.

In mathematics education these same arguments apply in relation to developing children's knowledge, say of arithmetical operations, and to developing skill in selecting appropriate operations, knowing *when* to add and when to subtract. Children are often taught to know *how* to add, subtract, multiply and divide, but may still be unable to decide *which* to do when faced with a real problem. Without experience of taking part in the construction of an algorithm in solving a problem the procedures have to be learned by rote, without understanding. They lack the understanding of what the process means in real terms and the ability to move from a verbal description of a problem, say of dividing a sum of money equally between a number of people, and the appropriate mathematical algorithm.

As in science, the aim in mathematics education is to stimulate and support learning with understanding. The interweaving of knowledge development with process skills and the resulting creation of models of the nature of things are elements that we believe should be at the centre of science and mathematics education.

This, then is the kind of learning which we aim to bring about in children through the education we give the teachers. To do this for teachers already in schools, through in-service education, it is often necessary to produce a quite radical change in their view of what teaching is and how children learn. This will not happen quickly or without some considerable effort on the part of those concerned. The *status quo* acts to moderate attempts at change and to establish new patterns and move to a new *status quo* takes a matter of years rather than the months over which in-service activities are usually spread. In the case of teachers in initial training the position is not so different, since these aspiring teachers will have spent up to 12 years in school through which they will have developed quite firm ideas about teaching and learning which may well have to be changed.

So, because of the existing ideas and experiences which teachers or student teachers bring with them, in teacher education we are concerned with changing ideas, not just planting new ones in virgin soil. Producing change is notoriously difficult. In the context of curriculum development it has been a matter of concern since the early days of curriculum projects, in the 1960s. In the 1970s, Kelly [4] and Rudduck and Kelly [5] carried out studies of implementation of innovation in which they distinguished between translocation (just getting new materials to teachers), communication (getting a message over), implementation (using the new materials) and re-education (developing real understanding and commitment to the new approach embodied in the materials). Their work has been followed by many other studies which have shown that producing materials and ideas alone is not sufficient to change practice and that this cannot be done without the active participation and cooperation of teachers.

The particular problems of science and mathematics.

Most of the points made above, although illustrated in terms of science and mathematics education, apply equally to most other areas of the curriculum. In science and mathematics, however, particular problems are encountered. Many, perhaps most, primary teachers have received from their own education a legacy of failure or at least dissatisfaction in relation to science and/or mathematics. Thus their overriding requirements are for confidence, an appreciation of the nature of scientific and mathematical activity and enthusiasm for teaching the subjects.

These pervading aims have implications for the conduct of a teacher education programme, since they concern attributes which cannot be engendered through specific content but only in the way of dealing with that content (which may mean, for example, not lecturing to a large group of students as the predominant style of a course). In meeting these needs it is as important not to do certain things as it is to do other things. For instance, it would seem important *not* to teach teachers the science and mathematics background they need in the same way as they were taught previously and which dismally failed. Neither should we underestimate the value of their everyday knowledge, which may be implicit, rather than explicit, but could be greater than assumed. Further, we should not treat them as if they had no existing ideas of their own about teaching, learning and about the subject matter to be taught.

Mathematics and science suffer from the popular perception that they are difficult, remote from the understanding of most people and only for the 'specialists'. Teachers, as members of the society in which these views are embedded, tend to share these perceptions. They stand, therefore, to benefit from actions which are taken towards creating a more positive popular attitude towards mathematics and science. More positive attitudes of teachers will influence the perception of these subjects by their pupils, the future citizens, and thus break into the present vicious circle in which unconfident teachers pass on their negative attitudes through the way they teach. Thus the moves to popularise mathematics and science are to be welcomed. For example, a study by ICMI [6] has provided both general considerations and concrete examples around the notion of presenting mathematical ideas of various level of sophistication to a wide audience.

A further problem particular to science and mathematics is that the majority of primary teachers in most countries are women. Like many women, they have suffered from the 'masculine image' of science and mathematics. By this is meant the reputation these subjects have of being 'cold and calculating', objective, concerned with facts and accuracy, impersonal and excluding emotions and feelings. Such characteristics do not, as a generality, seem attractive to girls, leading to a high rate of drop out and a sense of failure and alienation from these subjects. Many theories have been put forward to explain this situation, relating to the psychological origins of personality, in-born differences in spatial ability, social conditioning in early life, etc[7,8].

A growing body of opinion is looking at the nature of the subjects and the way they are portrayed in schools, rather that at the supposed deficiencies of girls, for the source of the problem. It has been suggested, in the case of science, for example, that "process-based science is likely to project a more human view of science and to involve learning experiences that engage the thinking, imagination and interest of pupils as well as leading to an understanding of key concepts and principles. The aim of this approach is for pupils to learn with understanding, through development of their own ideas, which are taken seriously and not ignored in favour of the 'right answer'. This type of learning is more likely to appeal to all pupils." [9] The same may be said of mathematics. If such a view of these subjects could be transmitted in teacher education it could play an important part in generating the confidence and enthusiasm which so many teachers lack. The importance of not reinforcing old prejudices follows clearly from this.

w.h.

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2 The Interface Where Science and Mathematics Meet and Mingle

Summary: Although mathematics and science have their own distinct disciplines of learning, they often "meet and mingle". Sometimes a mathematical problem may be evoked in a scientific context. At another time a scientific problem may be approached in a mathematical context. In other words, science often provides a framework, or context, for mathematical activity. Mathematics is often called "the language of science". The good teacher will be aware of opportunities offered when a scientific problem asks for a mathematical approach towards solving it. The same good teacher will also recognise the opportunity when science offers a framework for mathematical activity. Two examples are given, one from science and one from maths, that show how, at primary level, a simple intuitive approach towards problem solving can be developed into appropriate models, or general principles, which in effect are short-cuts towards solving more sophisticated problems.

Introduction

Science and mathematics are often mentioned in one breath, which indicates that people associate the one subject with the other quite readily. This is not surprising since many scientific observations can be quantified in numerical expression, in measured magnitude, or proportional relationship. Somewhere along the line of a scientific investigation, a switch is made from objective physical observation to mathematical processing of obtained data. The dividing line between the two subjects is crossed with ease, but is not so clearly drawn. Particularly in physical science, though not exclusively, one can benefit greatly from mathematical systematisation, logical consideration of possibilities, and clear questioning. Mathematical attitude and scientific disposition almost converge here.

It is little wonder, then, that there have been attempts to integrate science and mathematics as school subjects, in conversation or discussion as well as in intended practice, particularly at primary level. However, the conversation and discussion have proved easier than the implementation in practice. Where the attempt has been made to integrate the two subjects consistently, difficulties have arisen, emphases have been biased, and interests have clashed. Without a serious attempt to coordinate and align the differing subject matter of mathematics and science, the uneasy marriages have broken up all too easily.

Distinct disciplines, but ...

Science and mathematics are, after all, two distinct disciplines, each having its own characteristics. Using the word 'discipline' in close association with 'disciple' gives it the meaning of 'gathering information' or, rather, 'a process of building knowledge'. 'Knowledge' is then taken in the sense of a framework, or network, of interrelated concepts or ideas. The difference of obtaining and processing concepts and ideas distinguishes the resulting bodies of knowledge, in this case science and mathematics. The distinction, however, does not erase the obvious relationship between the two disciplines. Consider some examples where the two ways of gathering knowledge run together.

A mathematical problem may be evoked in a scientific context
Children take a daily measurement of their growing maize plant. However, over the weekends the school is closed and no measurements are taken. How much did my plant grow on Saturday? How much on Sunday? This is now a mathematical problem. Keeping careful daily measurements on the weekdays, enables the children to employ the mathematical skill of graphing these, so they can interpolate the most probable lengths of the plants on the weekend days. Alternatively they can use their measurements to calculate an average rate of growth, and use this per diem proportion to figure out the unmeasured magnitudes.

A scientific problem may be approached in a mathematical context
Where would my maize plants grow faster, inside the classroom or outside in the garden? An experimental situation as suggested by the question should be set up. Measurements need to be taken at regular intervals, both inside and outside, and the quantitative results compared. For more accuracy the daily rate of growth in both situations may be calculated and compared. For still greater accuracy a number of plants may be grown and measured, inside as well as outside, and average results may be determined and compared.

As these examples suggest, science often provides a framework, or context, for mathematical activity. The primary teacher who teaches the children both subjects ought to be aware of this, not so much from the point of view of designing a curriculum of mathematics, but in order to be ready to jump at every good opportunity to make the children apply their mathematical skill and knowledge in a situation of reality which happens to be meaningful to them. This places the children in a situation where they need and want maths. It enlivens the exercise, and provides sound motivation, for it carries within itself the reward of satisfaction.

Mathematics is often called 'the language of science', for it enables the young as well as the older scientist to generalise, to summarise and to communicate in clear and concise mathematical terms, formulations and equations. This is a great asset, and one more reason to insist on letting the children grow and develop in this most useful subject and apply it wisely. This simply means: teach the children mathematics with integrity and science with integrity. Instead of attempting to integrate what in essence stands apart, the two subjects are to be presented in a working relationship of interdependence where this is relevant and useful. In this way they become a most powerful educational 'tool' giving meaning and depth to the expression 'science and maths *education*'.

The use of mathematics as a toolkit for science makes it by no means subordinate to science, and certainly not when one considers that everybody's science would be severely crippled in its progress without mathematics.

Mathematics deals with quantities and magnitudes which are specific properties of physical objects and materials. Whereas mathematics systematically loosens itself from the bedrock of concrete, physical objects and their quantitative properties in order to pursue pure mathematical patterns and laws in the abstract regions of numbers, spatial relations and algebraic structures, science keeps returning to the reality of physical objects, their interactions and working systems. The flight of a scientist into mathematical abstraction is always related to some physical, down-to-earth, concrete, matter of fact observation, which poses a problem and asks for understanding.

Archimedes shouted 'EUREKA' because he found a mathematical construct to express (and find) the specific gravity of materials. Thus he could distinguish the specific gravity of the metal from which the king's crown was moulded. By comparing the magnitudinal properties of the pieces of matter Archimedes used a mathematical way to come to a reliable scientific conclusion with regard to the quality of the king's crown. His prediction was based on mathematical insight and physical experience.

Returning to the primary school, we find children who are taking their first steps in orderly scientific investigation. The essence of their scientific activities is 'encounter and interaction'. Children encounter (or are purposely confronted with) real objects, living or non living, which by their colour, texture, shape or behaviour invite or challenge the children to explore and investigate. This interaction between the children and the object of their immediate attention may be free and exploratory, but it can soon be given order and system by good science education when a good suggestion given by the alert teacher turns the free exploration into a purposeful investigation, whereby the children are encouraged and helped to develop and employ various scientific process skills. Observation, questioning, trying to explain, or hypothesising, predicting and verifying by experimentation, are readily mentioned as typically scientific process skills. However, quantifying data, measuring and related calculation or computing, belong to the category of scientific process skills, too. These, by nature mathematical activities, are now applied to the solving of scientific problems. Certain scientific challenges call directly for specific mathematical activity, which provides a motivation as well as an opportunity to initiate and develop, through practice, these mathematical skills.

Example: constructing a scientific model

These overlaps of maths and science should not go undetected by the experienced teacher, and the teacher in training should be given the opportunity to develop this detective eye, for this is the interface we are talking about. The following example illustrates where science and maths meet and mingle.

Strips of pegboard, with holes at equal distances, can be made into remarkably accurate balances. With sturdy paperclips as units of weight countless experiments can be done to find equilibrium. By simple trial and error combinations, of weights at various distances on either side of the balance arm, can be found which bring both arms in equilibrium.

However, when the teacher intervenes to help the children to bring order into their investigations, and to quantify their findings, the errors diminish, and the trials turn into direct experiments. A simple way of recording data is suggested whereby the relation between weight and distance on either side, or arm, of the balance is noted down in a systematic way. A worksheet with a series of problems like the one below is at the same time a model of good recording.

6W at ?D = ?W at 3D
= 2W at ?D

The question marks are to be replaced by the actual numbers (of distance to the fulcrum or of units of weight). If the balance is in equilibrium, it indicates that the figures are right. Initial trial and error is still in order. However, it will not take long before the children begin to suspect some relationship between weights and distances, and they use their suspicion, which by now has become a hypothesis, to predict the outcome of a possible combination of weights and distances on either side of the balance.

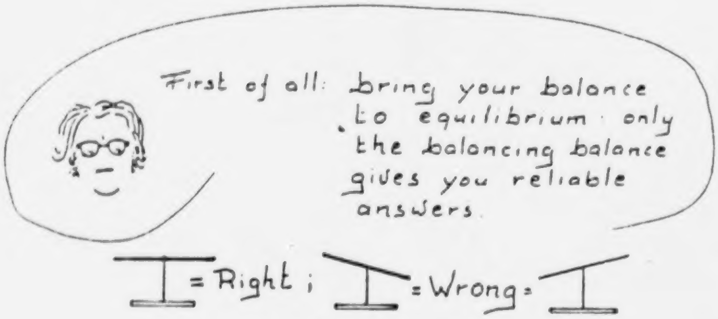
Slowly a pattern emerges which, often with some help of the teacher, can then be formulated and understood as:

> *The sum of Weights times Distances on the Left equals the sum of Weights times Distances on the Right.*

Once this mathematical construct has been mastered, the children can make countless combinations without touching the balance. Besides, this 'moments bar formula' enables them to find the unknown weight of other objects (pocket knife, pencil sharpener) by balancing them on their balance and then turning the algebra of the formula into a equation to be worked out by simple calculation.

The continuous interaction between science and mathematics is there in the primary school. In the case of the balance we find the science in the way the balance responds to changes in weight or of distance in an ordered and consistent way. There is a pattern in the way in which the forces, acting on either side of the balance, influence its behaviour. This pattern in turn can be expressed (or summarised) in a mathematical formula which applies in all physical circumstances. There is also an element of technology in this example of the balance. The instrument, simple though it is, must be made precisely and the weights must be cut to equal size, or volume.

The activities can be presented to pupils in a simple way. A series of 'worksheets' can set them going, giving the teacher a chance to pay attention to individuals, as the examples on the following pages may indicate. These examples were chosen from a unit called *Children and Balances.* [1]


First of all: bring your balance to equilibrium: only the balancing balance gives you reliable answers.
= Right ;
= Wrong =

These are examples of what to do and how to record it: Put 1 unit of Mass at Distance 14, on the left side, and 1 unit of M. at D. 14 on the right-hand side

	Left		Right	
	M	D	M	D
	1	14	1	$\boxed{14}$
	$\boxed{2}$	6	1	$\boxed{12}$
	2	4	$\boxed{1}$	8
1	1	14	2	☐
2	2	14	1	☐
			☐	4
3	☐	10		
	1	☐	3	11
4	3	☐	2	12

This is one problem !

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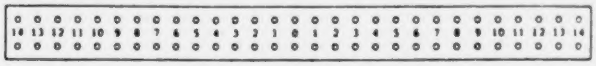
The "Law" of the balance
or What makes the balance balance ?

The following course of action can be undertaken by older children (age 12 - 81)

By solving some simple, direct problems - to which the balance "knows" the answers - you are led to a general conclusion: a generalisation, a "rule", a "law", which can be expressed in a formula.
(This process is called induction)

Understanding this formula enables you to solve new problems by deduction.

Provide enough balances with a peg-board strip as balance arm, suspended from the centre, top hole.



Number the holes as shown above
These numbers indicate the distances (D) measured from the centre (0). This is the fulcrum, or turningpoint.
(The amount of holes may be 14, 12 or 10.)

Use sturdy paperclips as "weights", as units of mass. One paperclip is: 1 M

Per hole you can use more than one unit of mass, of course; more than one paperclip

For instance, you are instructed to place "3 M at D8"
This means that you must place 3 paperclips in hole number 8.

Before you start, make a small rider out of bent wire, or a tiny paperclip, so that you can bring your balance arms into perfect equilibrium: only then can your balance give you faultless answers

Quantitative view

Developing a quantitative view of the world around is an objective of both science and mathematics education. This 'quantitative view' induces children (and adults) to find ways of measuring or otherwise quantifying things whenever this helps to solve a problem or whenever it would reveal some (new) relationship.

The mathematical requirements of the young primary scientist may be simple and straightforward, the necessary skills must well be mastered before any creative use can be made of them. Primary mathematics must take its own flight into more abstract working with quantities and

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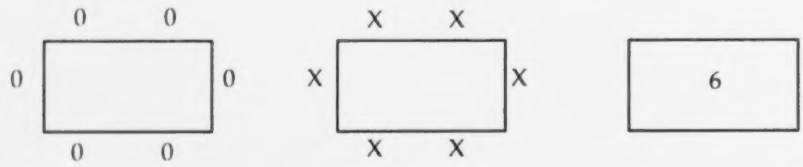
magnitudes simply because it has an educational value of its own. It offers a different, if not a wider, context of application than science.

Example: constructing a mathematical model

Constructing a scientific model of equilibrium with the help of simple experiences of balancing weights and measuring distances, which primary children can appreciate, was illustrated in the example given above. This finds its parallel in mathematical model building at primary level in the field of number work and algorithms.

Long division and its rules might have confused those of us who were, at one time, given these rules without supporting experiences. The following example attempts to illustrate possible steps to avoid this confusion by building a mathematical model which makes sense to primary children.

We can present the children with what may be for them rather a complex situation of seating 81 people at the occasion of a parents' meeting in the assembly hall of the school. There are tables available, each accommodating 6 people. How many of these tables will we need to make all 81 parents comfortable? In most cases the children would start by drawing a sketch of the situation and try and solve the problem by the means they have at their disposal: counting, adding, and trying to find short-cuts in their calculations. The problem may be approached at various levels:



$6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 3$

$\underbrace{6 + 6 + 6 + 6 + 6}_{30} \qquad \underbrace{6 + 6 + 6 + 6 + 6}_{30} \qquad \underbrace{6 + 6 + 6 + 3}_{21}$

$60 + 6 + 6 + 6 + 3$

So, at various levels, places are counted and grouped and added to reach 81 available places to match the 81 expected parents at a specific number of tables each seating 6 people at the time. Although they may use rather roundabout ways, the children will eventually find a solution to the problem. But it is all rather messy and disorganised.

So, next time a similar problem may be approached in a somewhat more sophisticated manner. A new problem is presented in the following way. "Freddy finds a box full of marbles in the attic. He takes the box outside where his brother Joe is playing with his friends Bernard, Hubert, Ed, Geoffrey and David. They are going to play at marbles, but want to start off with an equal amount each. How would they divide their treasure?"

This time the teacher places a box full of marbles on the desk, and leaves the children to divide this (as yet unknown) number of marbles fairly among the six boys. However, the process of dividing the marbles is now recorded onto the blackboard by the teacher, closely following, and thus describing, what happens:

Freddy	Joe	Hubert	Bernard	Ed	David
10	10	10	10	10	10
10	10	10	10	10	10
10	10	10	10	10	10
5	5	5	5	5	5
5	5	5	5	5	5
2	2	2	2	2	2

Three marbles are left over. Now they can talk about the sharing of the marbles and how fair it was. The figures on the board help them.

The next step is to move into a more theoretical problem: There are 324 marbles in a box to be divided among four boys. How many should each get? This is a problem in words without real boys or marbles, so in this case we must find another way to figure it out... Small worry: we can now follow the example of tabulating the numbers on the blackboard as it was done before:

	Peter	Bruce	Jim	John
324				
- 40	10	10	10	10
284				
- 200	50	50	50	50
84				
- 80	20	20	20	20
4				
- 4	1	1	1	1

No marbles are left over.

Some important questions have now been tackled:

- How many marbles were there?
- How are they to be shared?
- Are there enough (for each to get a fair share)?
- How many are left (if any)?

As a last step in this stage of *horizontal mathematising* (see Chapter 3, page 26) a problem like the following can be raised: "Linda has invited 12 friends to her birthday party. Before departing each one of them is to be given a bag of sweets. Mother has bought 425 sweets to be divided over the 12 bags. How many sweets will go into each bag?" This time the whole story is translated into numbers on the blackboard, and worked out in an interactive way together with the children:

	U	U	U	U	U	U	U	U	U	U	U	U
425												
-12	1	1	1	1	1	1	1	1	1	1	1	1
413												
-120	10	10	10	10	10	10	10	10	10	10	10	10
293												
-240	20	20	20	20	20	20	20	20	20	20	20	20
53												
-48	4	4	4	4	4	4	4	4	4	4	4	4
5												

... But who does not get tired of doing all this on the blackboard? The need to find a short-cut is now keenly apparent. From now on the **notation** scheme becomes the object of study, and the improvement on it makes the way to the solution of problems shorter and more efficient. This is *vertical mathematisation*. The problem might, in words, read as follows: Six girls divide 432 coloured beads among themselves for each to make a necklace. How many beads will each girl get? But... who, at this stage, needs girls or beads? The teacher will write numbers on the blackboard and will use only one pot now:

6 432
-60 6 x 10
372
-60 6 x 10
312
-300 6 x 50
12
-12 6 x 2
0

The "50 x 6", of course, is 5 x 10 x 6, which, being a sensible estimate, is a big step towards a short-cut in calculation. So, in the end, girls and sweets and beads and boys are left out altogether, and a purely mathematical construct, or model, or algorithm to work with remains. Now they can work out the answer to a mathematical problem on a more abstract level:

18 3866
-1800 100
2066
-1800 100
266
-180 10
86
-72 4
14 214 (rest 14)

This mathematical model now reflects fair division, and is understood as such.

These illustrations show how short-cuts in problem solving are looked for in both science and in mathematics.[2]

There are many instances where science and mathematics meet and mingle. So, when some science activity makes the children ask for anything mathematical, their interest is alive, and the never-to-be-missed moment has arrived, where the teacher and the children are to select the most appropriate mathematical tool. Recognising these interactive situations, and even setting them up purposely, belongs to the art of teaching and so finds a place of priority in the training of future elementary school teachers.

j.e.

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3 Children's Learning in Science and Mathematics

Summary: In the study of learning increasing attention is being given to the learner's activity. Activity here means mental activity not merely physical activity. It is important also to consider critically the notion of 'learning by doing', because the doing can be following small steps devised by the teacher or more creative exploration and interpretation stimulated by an open learning environment.

In this chapter, following a brief discussion of learning in general, the specific characteristics of learning in mathematics and science are considered. Similarities are found in the attention given to mental as well as physical activity, in using real contexts for problem solving, in the importance of communication, in starting from children's own concepts and in the importance of teacher intervention. As well as similarities, however, there are differences in approaches to mathematics and science education. Some of these exist because of real distinctions between the disciplines but others persist through tradition rather than present day reality.

Introduction

Learning and its theory have a long history. In this century the study of learning, initially mainly a concern of psychology, gained ground in the context of learning and teaching in school. A prominent theme in this history is an increasing emphasis on 'activity', that is, on the activity of learners. In science and mathematics education the emphasis on activity, on 'learning by doing', stands out particularly.

In the next section the key concepts of active learning will be identified through consideration of learning in general. Then these key concepts will be our guideline in elaborating children's learning of science and mathematics. Designing instructional materials and teaching in both fields, making connections between the subject areas but also keeping them distinct, requires knowledge of both similarities and differences.

The study of learning in a wider context

The behavioural psychology which dominated the study of learning at the beginning of this century has given way to cognitive psychology which emphasises the role of mental activity, as opposed to unthinking 'response' to stimuli, in determining behaviour. At the same time it has been recognised that studying learning in controlled laboratory conditions gives little information about learning inside and outside schools. Gradually the view has emerged that learning depends upon what is learned, in what contexts and with what motivation. So educational psychologists have taken a position closer to the disciplines and to school subjects and as a consequence closer to pupils and their learning activities.

The recognition of the role of the activity of the learner led to the notion of learning by discovery, the desire to give pupils the excitement of finding things out for themselves. In science this approach, also known as the heuristic method, was advocated at the end of the last century, but was only widely adopted in the 1960s. Its deficiencies in practice were soon apparent. It was extremely difficult for pupils to arrive at accepted generalisations through their own observations and investigations. Thus the notion of 'guided discovery' was introduced, giving the teacher a role in structuring the learning situation. However, a more fundamental criticism of discovery learning, whether guided or not, was that it makes no explicit reference to pupils' own ideas.

The notion of activity in learning, implicit in 'discovery', also appeared from Eastern Europe as well as from the West. But it was highly structured and left little room for learners' own constructions. Western educators and learning theorists stressed personal involvement in the learning process more and more; experiential, participative and cooperative learning became popular terms. Particular value was placed on problem solving, using realistic and relevant problems.

But in practice it was soon realised that 'activity' of itself is not necessarily accompanied by learning. What could be done to transform an activity into a learning activity? One of the convincing answers came from the designers of 'Problem Based Learning' curricula [1] in which students create their own learning activities starting from a relevant problem. In the initial steps relevant prior knowledge will be brought to bear in tackling the problem and will be restructured as new knowledge is acquired. Throughout the whole process time is spent on reflection. Working in cooperative task-group stimulates interaction in which individual constructions can be shared. The necessity to put mental images into words appears to support reflection and arguably raises the standard of thinking.

Children's learning in science and mathematics

The common thread in a large number of studies of learning in science in the last two decades is that children bring to their new experiences existing ideas formed as a result of earlier experiences, formal and informal, processed by their own ways of reasoning. These ideas make sense to the children, often more than the accepted scientific views of things, which they thus reject. The recognition of the existence and nature of children's own ideas led to the realisation that it was frequently ineffective simply to attempt to teach the 'right' concepts. Attention thus turned to different ways of teaching which took account of existing ideas. Several strategies have been proposed. An early and still popular one is to introduce an event or phenomenon which is discrepant or conflicting with the pupils' view in the expectation that this will cause a modification in thinking.

At a theoretical level, Piaget's notion of provoking disequilibrium in order to bring about accommodation of the mental framework to encompass new experience suggests that new experiences should challenge existing ideas. However, there has been much discussion of the nature of the dissonance between the pupils' ideas and those required to understand the new experience. Too small a gap means that pupils assimilate the new experience into existing ideas (perhaps with minor modifications); too great a gap means that the new experience makes no connection with existing ideas which are left unchanged. The different ways proposed for using 'discrepant events' in the classroom include following the event with groups discussions [2], brainstorming and then debating ideas [3], charting all the ideas coming from the class [4], and 'interpretive discussion' [5].

There have been warning notes, however, about the effectiveness of discrepant events in practice, since children may be less concerned about having their ideas challenged than adults would be. In addition an objection on more ideological grounds suggests that the notion of a conflict between ideas and the subsequent decision as to which one 'wins' is unsound as a basis for learning. Certainly it would not seem to aid pupils' ownership of ideas. It can be argued that these objections have particular force at the primary level, where it is important to take children's ideas seriously. Finding out what children's ideas are in order to 'confront' them is not the same as requiring children to use and test their ideas, as a result of which the ideas may be modified or perhaps abandoned in favour of ones which *they* decide are better fits for the evidence available.

This process bears close resemblance to the way science has been constructed during the history of mankind; mathematics too was constructed likewise in different cultures and in different places. In the so called 'genetic' approach of teaching the historical development of the subject, the ontogenesis, students are offered opportunities to 're-invent' what mankind invented earlier [6]. Materials for learning and teaching in the domains of science and mathematics have been designed with this genetic approach [7].

In the current view of learning in science and mathematics the core concept is activity [8] and, according to modern science and mathematics educators, learning is 'learning by doing science and mathematics'. Learning by doing is the device but by no means the whole story.

What students do should be intrinsically motivated, perhaps through presenting a realistic problem-situation and an opportunity for investigation, cooperative and interactive, using prior knowledge. Learning by doing requires reflective thinking and creates opportunities for personal constructions. What has to be learned ought to become a personal mental property, integrated in what was acquired before. It becomes, so to speak, 'owned' by the learner.

Together with knowledge and skills in the field of science and mathematics, pupils acquire a specific attitude towards identifying, tackling and solving problems. It is this attitude that supports continuous learning, even in situations a long way from classrooms or when the schooldays have been left behind. Further, by doing science and mathematics students will develop a very specific and personal view of these disciplines. The approach reflected in this publication will embrace a concept of science and mathematics as a process, in contrast with science and mathematics viewed as a collection of rigid facts, procedures and rules.

Science, as portrayed by prominent philosophers, is seen as the construction of explanatory models that encompass wider and wider ranges of phenomena. This is not all that different from the learning of science seen as the construction by pupils of ideas about the world around and tested against their own experience. Thus these two advances are regarded as connected and as leading towards a new science of learning with great import for the learning of science [9].

Points of special attention

Having traced the roots of 'learning by doing' in educational psychology, we now use the identified key concepts to look more closely at learning in science and mathematics.

Activity

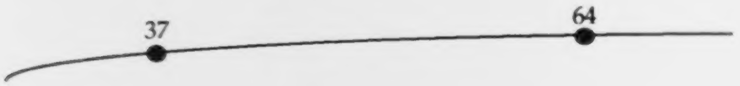
On a variety of occasions environments change into learning environments. It can be an event, a problem, a phenomenon or an argument that raises questions and asks for investigation.

The situation can be mathematical by nature, in which case investigation means mathematising, horizontally first (in order to put the problem into a mathematical context) and then vertically (using mathematical tools). (See chapter 2, p 20 for an example of these types in action) . Other important activities include organising, describing, mapping, using suitable schemes and models, ordering the raised questions and systematically searching for answers.

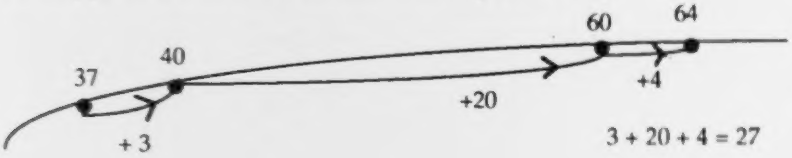
For example the 'scheme' of the empty number line has been found very helpful in assisting mental arithmetic. Take the following problem, to be solved by pupils in grade 3 [10]:

> A book has 64 pages.
> I have read 37 pages.
> How many pages are left for me to read?

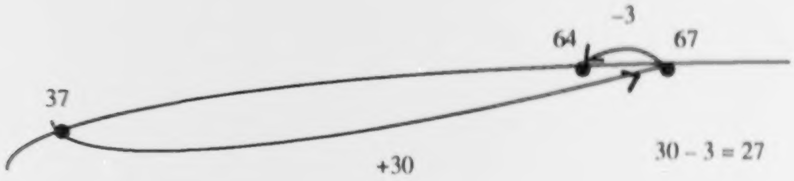
The line is a thinking model to represent the book:



One approach is to work forwards in steps, this way:



Another is to go beyond the end to an easy point, which does not exist in reality but can be considered as an extension of the line:



In this kind of activity prior knowledge and intuitive notions are used and new knowledge and notions are developed. The need for being efficient supports the invention of short-cuts, a way to establish mathematical procedures like algorithms.

In science, 'activity' is characterised by the use of process skills (see Chapter 1, p7), observing and detecting, raising questions, formulating hypotheses, making predictions, and reporting the findings, simultaneously communicating and discussing them. An example of these skills in action has been given in Chapter 2, p15.

Realistic situations and contexts

Realistic means realistic to pupils. In other words it is not just daily life, but play and imagination are also realistic for (young) children. Context means what comes into mind because of a situation: intuitive notions, pre-concepts, misconceptions, experiences, (ir)relevant memories, successful trials, a nice solution, an unforgettable failure etc.[11]

Realistic situations and contexts are essential conditions for the learning of science and mathematics because:

- applications are met, learned and practised from the very beginning

- well known situations motivate, pupils recognise usefulness
- in realistic situations prior notions and pre-conceptions are easily brought in
- some contexts become exemplary and can develop into general models[12].

It is not easy to create realistic situations designed to become learning environments which meet these conditions. Whilst the imagined division of a pizza appears to offer a good orientation on fractions, and a yoghurt-cup telephone can provide a useful exploration of the phenomenon of sound as a vibration, it is more problematic to introduce 'kinetic energy' through the realistic context of 'windmills generating electricity' and in a context of 'banking' the multiplication of negative numbers will not become clear. There is often a conflict between the detail of the reality and the requirements of the disciplines.

Interaction

We may say that 'activity' in the field of science and maths becomes 'learning activity' if, among others, fundamental notions are constituted, concepts are developed, new views arise, skills are practised, procedures are developed and models are acquired. Just doing is not sufficient: what has been done needs to be put into words. This activity of formulating and expressing their ideas needs reflection and anticipation; reflection on their own learning process and anticipation of how other learners need to be told to understand what they have learned. Working together with other learners creates the necessary stimulation. Interaction is more than communication; pupils learn to understand each other, they learn to listen, to immerse themselves in the thinking of peers and teacher, to feel for others' efforts and to realise that they must give access to their own thoughts. Meanings that initially have been constructed individually will be shared and completed by interaction.

Personal constructions and productions

Science and mathematics bring worthwhile knowledge and skills if (and only if) these become an integral part of an individual's 'common sense'. By common sense we mean the approach which makes sense to the individual and which s/he uses to tackle problems in daily life. Common sense can be developed to various levels and on this depends the extent to which the subject matter, understandings and skills can be used.

Research and practical wisdom show the inadequacy of school knowledge which exists only in artificial settings and remains isolated from everyday applications. Children cannot use this knowledge and so often revert to more primitive procedures, such as counting instead of multiplication. The procedures used go back to pre-school learning and are so to speak 'true to nature'; sometimes similar to those which can be recognised in primitive stages of the ontogenesis of the discipline. The research literature on 'ethno-mathematics' provides examples of this phenomenon and suggests that pupils' intuitive and natural approaches to problems[13] are a worthwhile starting point for learning. In mathematics a similar ontogenesis has been found in different locations and in different cultures[14]. It goes without saying that children should be enabled to construct their own (primitive) notions and procedures, as happened in the history of mankind, of course, without imitating the big blunders or simulating the long periods of stagnation. *Reinvention* is the term used; children are stimulated to invent their own mathematics.

Here is an example of a reinvention by a child in tackling this subtraction sum:

$$\begin{array}{r} 324 \\ \underline{-187} \\ 263 \end{array}$$

3 minus 1 equals 2
2 minus 8 equals 6 short
4 minus 7 equals 3 short

200-60-3 = 140-3 = 137

Most interesting are children's free productions as answers to 'classroom tasks' such as:

- make a manual for this calculator to be used by pupils of grade (n-1)
- think of a test on doing this for your friends
- write a book about the number 1 million
- invent all kinds of sums with the answer ($^3/_4$)
- tell younger children how to graph the growth of a bean
- how would you explain that a big piece of styrofoam is lighter than a small stone?
- devise a test to find which (of three given samples of) paper would be best for covering books?

Children create their personal science notions as well[15]. As in mathematics the teacher has to intervene so that pre-conceptions are adjusted through checking 'common sense' ideas against the evidence of nature.

It seems to be evident that in both subjects 'learning by doing' needs teacher's support and intervention. Core questions posed at well chosen moments stress the essentials. In maths, for instance, the suggestion to use a particular scheme or model can open doors. The skill of reporting accurately (orally in discussion, as well graphic, in writing and diagram, graph or sketch) becomes important here.

So it is not only the investigating activity which needs support; important stages need to be concluded by reflective summarisers or to be anticipated by advance organisers and specific tasks have to be given in order to memorise and practise skills.[16]

In science any of a range of interventions may be used. For example the following have been proposed by the SPACE project [17].

(i) ***Enabling children to test their own ideas***

This will involve children in using some or all of the process skills of science: observing, hypothesising, predicting, planning and carrying out fair tests, interpreting results and findings, communicating. It is an important strategy which can, and should, be used often. Implicit in the suggestion of **using** process skills is the notion of **developing** them, for example, through greater attention to detail in observing, more careful control of variable in fair tests and taking all the evidence into account in interpreting.

(ii)***Encouraging generalisation from one context to another***

Does an idea which has been proposed for explaining a particular event fit one which is not exactly the same but which involves the same scientific concept? Other contexts might be suggested by the teacher or by the children. The application may involve discussing the evidence for an against, or gathering more evidence and testing the idea in the other context, depending on familiarity with the events in question.

(iii) ***Discussing the words children use to describe their ideas***

Children can be asked to be quite specific about the meaning of words they use (whether 'scientific' or not) and to provide examples in action where possible. They can be asked to think of alternative words which have almost the same meaning. They can discuss, where appropriate, words which have special meaning in a scientific context and be helped to realise the difference between the 'everyday' use of some words and the 'scientific' one.

(iv) ***Extending the range of evidence available***

Some of the children's ideas may be consistent with the evidence presently available to them but could readily be challenged by extending the range of evidence. This applies particularly to things which are not easily observed, such as slow changes, or those which are normally hidden, such as the insides of things. Attempts to make these imperceptible things become perceptible, often using secondary sources, helps children to take a wider range of evidence into account.

(v) ***Getting children to communicate their ideas***

Being required to express ideas in one way or another – through writing, drawing, modelling and particularly through discussion – means that they have to be thought through and often rethought and revised in the process.

Differences

Having mentioned a number of similarities between learning in mathematics and science, we now turn to the possibility of differences. A first difference comes up when we consider the way a situation is organised for solving problems in mathematics. As was shown in Chapter 2, in relation to the creation of an algorithm for division, the organising of the sharing situation became a organisation model for doing future divisions and eventually it becomes a kind of mental model to do the calculations without the need for further reference to real things. In science, however, the eventual check is always against reality, not internal logic, as it is in mathematics.

A second difference is that mathematics distinguishes itself from science in respect to the concept of 'being certain'. What gives direction to the process of problem solving in mathematics classrooms, is the search for certainty.

> Seven persons go into a double decker bus. Some are going upstairs, some are staying downstairs. How many possible distributions can you find?

There is a certainty implied here that there is indeed a single solution and that no-one can find a greater number of distributions. This specific kind of certainty is missing in science investigation, where the possibility always exists of an alternative answer which fits the evidence.

But the validity of differences between the subjects depends to some extent on whose judgement is being used. While it is clear for the general public that there is rapid development at the frontiers of most sciences, the same can hardly be said about mathematics. There are possibly many roots for such a misconception. On a somewhat anecdotal level, the mere fact that there are Nobel Prizes related to various sciences, but none in mathematics, may suggest to the lay person that nothing 'new' is happening in mathematics. Perhaps more to the point is the fact that while the vast majority of today's adults will have learned mathematics for many years in the classroom, the mathematics they have then encountered is for the most part centuries old, be it in arithmetic, geometry, algebra or even calculus. In contrast to the science teacher who can find many occasions to relate aspects of the curricula to recent developments, the mathematics teacher is usually working with a curriculum that can only reinforce the image of mathematics as a static discipline.

Implications

For the general public, mathematics is concerned essentially with calculations and formulas, a view strongly supported by both the standard primary and the secondary school curricula and related to the traditional conception of mathematics as the science of number and shape. The recent developments in electronic technology have created a situation which may suggest to the public that a shift in emphasis could – and should –

happen. While in certain historical or sociological contexts the ability to perform, say, arithmetic calculations might have rightly been considered a high-level skill, this is no longer the case, now that calculators are available – almost universally – to evaluate these routine operations.

So emphasis in the primary school curricula must change, for instance, from calculating skills needed for the exact evaluation of more or less complex arithmetic expressions to the decision-making skills enabling one to chose the appropriate arithmetic operations corresponding to a given situation and to assess the reasonableness of the answer. In other words, a shift from purely algorithmic skills to more complex interpretative ones.

There has been recently a strong movement to revamp the public perception of mathematics by presenting it not merely as the science of number and shape, but rather as the *science of structure, of order, of patterns of all sorts* [18]. Such a view, while not negating the fact that numbers and forms are at the very heart of mathematical activities, clearly stresses the idea that it is not the ability to manipulate such mathematical objects that is crucial, but rather the ability to use them in a proper way. Much of the computational drudgery can now be safely left to the calculator (or the computer), which allows for more attention to be given to the mathematical process *per se*.

Developing skills in the execution of arithmetical algorithms has always dominated the traditional primary school curriculum. It is probably safe to say that any algorithm of such basic importance that it should be included in the primary (or even secondary) school curriculum will have been programmed and made available on computers (if not on calculators). So placing the sole emphasis on performance in algorithmic computations is definitely not the best way to prepare the pupils adequately for the mathematical needs of the twenty-first century.

Does this imply, say with respect to the primary school arithmetic curriculum, that there is no more a place for the study of basic calculation techniques? Surely not! But the education process should promote the acquisition not of 'mechanical' abilities, for which the machine is superior but of more 'human' abilities pertaining to the choice of an appropriate mathematical model, the planning of the operations, the development of *number sense* allowing one to check, through mental approximation, the order-of-magnitude of the results (not the exact value!) and to interpret those results intelligently. All this results in quite a different agenda from the traditional goals of primary school mathematics. While the exact nature of the learning experiences needed for the development of the new skills is still to be assessed, such skills will surely require a thorough understanding of the fundamental principles on which is built the new practice of mathematics. The key issue does not thus concern whether the teaching of fundamentals is to be included in the primary school curriculum, but rather *which* fundamentals should be included and *how* they should be presented to the pupils.

This leads us into the next chapter where we consider the knowledge that teachers require in order to produce this learning in primary pupils. Later, in Chapter 7, the implications for training are spelled out.

f.g. with w.h., b.h.

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4 What Should Teachers of Primary Mathematics and of Primary Science Know?

Summary: This chapter explores how teachers should develop an intellectual structure of interrelated concepts on which they can draw with confidence when placed in a problem situation. They interact with the children as well as with subject matter, and face problems concerning both. The chapter outlines five distinct aspects of knowledge which the teacher should cultivate in order to be well equipped for his or her profession.

Introduction

"Teachers should know more than the children they teach". This is the simplest, the quickest and the most meaningless answer one can give. Although it sounds like 'common sense', it requires distinguishing degrees of knowledge the nature of which remains obscure. Having followed a more advanced course in mathematics or science or, indeed, having passed an exam at a higher level, is of little or no relevance unless it results in teachers having ready, usable knowledge at their fingertips. In other words, we discuss not what they should have had, neither what they should have passed, but what they should have made their own: *an intellectual structure of interrelated concepts on which they can draw with confidence when placed in a problem situation.* This knowledge concerns science, mathematics and education. It includes the disciplines of the subjects, the subject matter, as well as the complications of helping children to learn. Altogether this forms a many-faceted construction, which is considered further in Chapters 6 and 7.

A triple interaction

Children learn best when they occupy themselves fully with the 'things' that make up the subject of their learning. In science education these 'things' are real, concrete, touchable living and non-living objects or materials which are found or placed in a variety of situations. It could be a seedling growing on blotting paper and another one growing in soil. It could be a magnet picking up pins. It could also be a small community of living organisms growing, or moving about on a square metre of ground (a 'minifield') at the edge of the garden or somewhere in a piece of wasteland, or it could be a mealworm in a box given a choice between heaps of sand, sawdust and cornmeal.

In mathematics it could very well mean the same things, but then in some quantitative relationship on a slightly more abstract level: e.g. the daily rate of growth of the seedlings; the number of pins the magnet picks up; the measuring of equal distances between the mealworm and the three heaps of its choice; or the different sets of living things found within the 'minifield', or their coordinates for placing them on a map. But even if the 'things' of maths occupying the children are of a more abstract character, such as the symbols they use for numerals and signs of operations, or the planimetric figures they draw, these are still a self-created reality with which the children work.

In occupying themselves with their subject matter, be it the very concrete things of science, or the partly abstract symbols of maths, children go beyond mere encounter. They enter into a 'dialogue', an interaction with the object of their immediate attention, and they manipulate it in such a way that it reveals something of its own essence. This may be a property, or a 'way in which it works', or a sequence of 'things I can do with it', or something that has been uncovered, or that has been invented or even re-invented.

The 'revelation of something of its own essence' is nothing else but the formation of a concept in the mind of a child. This may be a new concept altogether, or some change or refinement of an already present concept. It could also be the rejection of some preconception which is now suspected as false. Finally, it might be a new association with some previous concept(s) so that a relationship is established which deepens understanding and insight.

The teacher who wants to engross the children in the subject matter of science and mathematics through real things and representative symbols which the children can handle, must himself have undergone, or undergo, the excitement and joy of learning, and so share it with the children. Only then can the teacher appreciate a learning situation, assess its value, judge what next step should be taken, create new learning experiences, and foresee where the renewed investigation can lead.

Foreseeing what a learning experience can lead to implicates an interaction of the teacher with the subject matter at hand on the level of an adult, someone who has learned, through experience and study,

a) to place a new concept in a wider framework of existing knowledge;
b) to use a familiar concept in the wider context of one's (own) existing knowledge;
c) to retrieve an old concept and to see its relevance in a new situation.

With this more mature intellectual potency the teacher can approach both the subject at hand and the querying, learning children, and so interact with them, being sensitive to their needs or wants, and helping them with word as well as with deed, so that they can see relationships and understand an explanation. The teacher's interaction with the children and with the subject matter may assist the children to recognise and make use of a relationship which they had learned already, so that they find or discover their own explanation of what they were puzzling about, or gain a new insight.

Now we can summarise the tripartite interaction which is in evidence in good education:

1) The child interacts with the 'subject matter' at hand (e.g. germinating seeds, darkness seeking woodlice, an isosceles triangle, or some quantity to be divided). At the same time the child interacts with the teacher who with word, deed, or other intervention (or, indeed, by just leaving the child quietly alone) helps the learning.

2) The 'subject matter' (which, broadly taken, comprises the 'stuff to be learned', the things to be handled, the sources being tapped, and the problem situations into which the children are placed) interacts both with the children to whom it presents a challenge, and with the teacher to whom it once was, or may now still be, a challenge.

3) The teacher, in turn, interacts with the querying children as well as with the object of their query, namely the 'subject matter' at hand.

It is with this triple interaction pattern of education that we should seek for what teachers of primary mathematics and science ought to know in order to be good and confident in their teaching[1].

Knowledge

The word 'knowledge' does not simply represent a clear, unambiguous and unequivocal concept. By different people, or in different circumstances, it may be given various shades of meaning.

Popularly stated we distinguish:

1) A *'How-is-it-called-knowledge'* which relates mainly to the formulation of factual information, the right name, the appropriate word. It helps to show that you know what you are talking about, yet does not guarantee it. Learning by rote tends to lead to this form of 'knowledge' and go no further.

   Because this aspect of knowledge is easily assessable, it may be associated with what is referred to as 'school knowledge', and as such be given undue desirability.

   Yet we should not underestimate the value of 'communicating knowingly' as it creates permanence in one's own mind as well as intelligent intercourse among communicating humans. Perhaps we ought to refer to this aspect of knowledge (which for a great deal fills our intellectual 'archive' called 'language') as 'social knowledge' since it covers words and terms, conventions and rules, established and agreed upon by society, which we can only learn by social contact with others by word of mouth or, otherwise, in writing.
   As examples one could mention: 'Naming the parts of the flower', or 'Naming the internal and external features of the frog'. Nomencla-

ture is an obvious example. In mathematics one could think of the names of counting numbers, the names of geometrical figures, or of algebraic symbols.

2) A *'How-it-appears-to-be-knowledge'* which refers to direct experience. It is a knowledge emerging from interaction between an observer and the object under observation. It embodies properties or observable behaviour of real objects and their possible or actual interactions with other objects, or with the observer/learner. It is checkable knowledge, based on experience, observation, experimentation, research, induction. This makes it also changeable knowledge, adaptable, in as far as it can be adjusted by new experience or investigation, revealing new evidence. Therefore, it is often imperfect, biased, open to discussion. It may also possess a certain degree of probability.

   This knowledge makes 'prediction' possible: it stretches ahead. When you see lightning, you can expect a thunderclap; when multiplying any number by two results in an even number. Since the evidence supporting this knowledge is searched for in the physical world — in the nature of things as they are — it may, perhaps, better be called 'physical knowledge'.

3) A *'How-does-it-relate-knowledge'*. This conception signifies a higher order of abstraction in which relationships between concepts become evident: generalisations, conclusions, and patterns inferred from repetitive experiences or experiments. It may also be deduced from pure thought processes in which existing concepts, at various degrees of abstraction, are related to each other, or are recognised as related to each other. Even reflecting on such thought processes may form new knowledge which, in the sense here elaborated, comes close to being identical with 'insight' or 'understanding'. A better name to indicate this knowledge would be: 'logical knowledge'. It is at this level of knowledge that we talk of insight and understanding, because it has been 'processed' to fit into a person's total store, or 'web' of acquired knowledge, beyond pure experience undergone at first hand.

4) A *'How-it-should-be-done-knowledge'* which comes close to an ability to do things. However, it goes beyond the physical ability or skill in as far as it incorporates not only the remembered sequence of operations, but also the foresight to create, to invent, to lay out, and to plan a succession of processes or operations in order to compose things, to put things together, to calculate, to compute, to program, to experiment, or to run an investigation. It has to do with algorithms, with rules of thumb, with safety precautions, with operations, physical as well as mathematical. This knowledge precedes skills and abilities, and motivates to do the necessary exercise to

acquire these. One could call this 'technical knowledge' but then in a more than purely mechanical sense. Above the levels of imitating and the grasping of procedures one needs insight and inventiveness in order to be able to organise an experiment or an investigation. Resourcefulness belongs to this aspect of knowledge.

5) Finally there is a *'How-to-find-your-way-about knowledge'* which makes a person resourceful enough to tackle any problem in the most effective way possible. (Tackle, not necessarily solve!) It consists of a knowledge of resources and their accessibility. It is the knowledge of how to make use of any level of existing knowledge within oneself, coupled with the knowledge of where to look for and obtain the new information, advice, or resources one needs. One knows where to look for things, situations, people, books, scripts, charts, maps, archives, tables, computer ware and whichever other modes of storing information may be useful. This is the knowledge which leads directly to self-reliance in searching and in learning. It is the knowledge of the professional and could, perhaps, aptly be called 'professional knowledge'.

Distinctions, not levels

It should be emphasised that the five distinctions described above are not five different kinds of knowledge, nor even levels of knowledge. Knowledge is an intellectual relationship of a person with the world of thought and reality, and is in this person one and undivided. The actual development and actualisation of a person's knowledge may, however, be biased towards one or more of these five aspects of knowledge. This may enhance or impair its quality with regard to insight and understanding (of, for instance, mathematics and science) and so determine its use or usefulness.

The knowledge which a primary teacher (of science and mathematics) **should cultivate**

Without going into details of the content of possible maths and science courses in primary teacher training institutions, we can indicate and state that one should strive for as close a balance as one can accomplish between the five outlined emphases, or aspects, of knowledge. In mathematics as well as in science the teachers should become fluent in the appropriate language and use of symbols. They should be rich, and become richer still, in direct experience by a very active and totally involving (workshop) approach. Through reasoning in discussion, reflection on action and thought processes, and wide related reading and discourse, they should gain in comprehension, so that they build up a framework of insightful understanding on which they can rely in their work as well as in their further personal development, with confidence.

Successful teaching of children in mathematics and in science requires a knowledge of the basics, the elements of the subjects. This knowledge requires a deeper understanding than many teachers seem to achieve in their training courses. This seems to indicate and imply that the elements of the primary school's subject matter refers to what lies at the foundation of the sciences, and these foundations are dug deeply. 'Elementary' is therefore by no means to be identified with 'childlike' or 'easy'. These elements, then, are to be pursued in depth in the training courses, rather than superseded by 'more advanced subject matter', whatever that may be.

Apart from having 'necessary' knowledge it is important for the teacher to be aware of having a distinct view of the subject. Is science an accumulation of facts to be conquered by gathering factual knowledge? Or is science itself a way of learning: the operative skill of acquiring knowledge by the intellectual processes of observing, questioning, hypothesising, experimenting and reasoning? An answer to either of these questions will express the teacher's view of science and profoundly influence his mode of teaching the subject. Many people tend to accept that acquiring knowledge of the facts of science is the purpose of science teaching, and would consequently emphasise the orderly organisation of these facts in graded packages for learning, if necessary by rote, but supported by demonstrations and illustrations, according to the assumed intellectual capacity of children in different classes. However, the view of science as an intellectual process of active learning is adhered to by a growing number of educators, and this results in a way of teaching which we could characterise as 'problem posing teaching promoting problem solving learning'[2]. Similarly, the teaching of mathematics requires some flexibility of approach where mathematical processes enter the picture. For instance, in teaching computation there is more than one way of familiarising the children with algorithms. If teachers tend to teach only 'their own' algorithms, they should become aware that there are many different models, some of which may be invented by the children. The consideration and conscious appreciation of a different algorithm, or indeed any other mathematical model, reaches beyond pure computation and calculation, and helps the children really to understand mathematical processes. The example of constructing a mathematical model given in Chapter 2 on page 19 illustrates how this kind of mathematisation can occur in a primary classroom.

It is insight and intelligence that should be challenged in the future teacher. It should touch upon all the facets of the teacher's knowledge that we have emphasised above. It should not be unduly overbalanced and directed to one particular aspect, as often happens in rote learning. Teacher training courses that move away from the kinds of knowledge that matter leave students bewildered and battered, sighing: 'This science (or this maths) is not for me', in which case neither will it be for their children.

Teachers need delight and enthusiasm in their own learning, their own formation of knowledge. By being totally immersed and involved in the process of 'making' their own basic knowledge of science and mathematics, they learn to help the children work toward making their own knowledge, too, for there is truth in the remark that teachers tend to teach the way they themselves were taught. Let either be appropriate and efficient.

j.e.

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5 Education for Assessment as Part of Teaching

Summary: Assessment of the ideas, skills and attitudes which learners bring to new experiences is central to teaching for understanding. It is essential, therefore, to be able to assess the full range of process and product outcomes of learning. However, this chapter has to be selective. After a brief discussion of features and various purposes of assessment, attention is given to assessment as part of teaching and particularly to those aims of active learning which are not amenable to conventional assessment methods. It is suggested that indicators of process skills and attitudes, devised by teachers in workshops, are used in gathering and judging information about children's activity in a systematic way. Mention is made of the value of involving children in their assessment and of self-assessment for teachers.

Introduction

The model of decision-making in teaching, proposed in Chapter 1, suggests that assessment, carried out in a manner which is consistent with the teacher's views of the subject matter and of learning, provides feedback as a basis for adjusting pupils' learning experiences. An interaction between the views of learning and of the subject, the decisions about classroom activities and assessment was thus identified. In theory the assessment procedures follow from the decisions about learning experiences which are related to the curriculum goals. However in practice the situation is often different and the assessment procedures come first and may dominate what is taught and how it is taught.

The extent to which the assessment leads the curriculum rather than vice versa depends upon the importance given in a particular context to assessment and testing in primary education. It will be affected by whether there is an end-of-primary school examination and whether progress from one year to another is determined by end-of-year tests set within the school. If assessment has a high profile it will tend to lead the curriculum in all subjects which are assessed. Those subjects *not* assessed are likely to be neglected in all but lip service. In these contexts, the success of any innovations will be limited by existing procedures for assessment and testing. Thus it is necessary to develop appropriate assessment procedures relating to any innovations of curriculum content, methods or goals. It follows that training in these procedures has to be included in teacher education courses, otherwise teachers will fall back on established assessment methods and will adjust their teaching to these.

It cannot be emphasised too much that appropriate assessment methods have to be devised and made available to teachers if there is to be any chance of implementing changes in children's learning experiences. Developments in the last few years have meant that it is no longer acceptable to claim that process skills and understanding (as opposed to knowledge of facts) cannot be reliably assessed; the techniques are now widely known. Indeed it is becoming accepted, perhaps reluctantly, that to change assessment is perhaps the swiftest way of bringing about change in teaching. It was certainly in this way that process-based learning was given a firm foothold in secondary science in schools in England.

This is not the only argument for giving new procedures in assessment a prominent position in teacher education courses. There are many other reasons, perhaps the most important being that assessment is an essential component of putting into practice the constructivist approach to learning that is endorsed in this document. It is possible that some teachers do not recognise the role of assessment in their teaching, or regard it as too informal and not 'proper' assessment, which they see as formal and separate from learning activities. A change of attitude towards, and understanding of, assessment purposes and procedures is signalled for these teachers.

Meanings and roles of assessment

The view of assessment that we are adopting here is that it is a process in which information is collected about performance, compared with some standard, criterion or expectation and a judgement is made on the extent to which there is a match. In this view, assessment results in a record or response which replaces the actual behaviour (which can only be preserved if the pupils' work is preserved or video-recordings are made). It follows that there is always some selection and some information is lost. Results of assessment are not 'the real thing' and cannot tell the whole story. This must be kept in mind in interpreting and using the results of assessment.

The various ways in which information is collected and the various bases for judging it create the variety of different kinds of assessment. These include standardised tests, where information is gathered whilst children are tackling carefully devised tasks under controlled conditions and, in contrast, ongoing assessment, carried out almost imperceptibly during normal interchange between teacher and pupils.

A major distinction is to be made between tests (and examinations) and other forms of assessment. Tests are specially devised activities designed to assess knowledge and/or skills by giving precisely the same task to pupils who have to respond to it under similar conditions prescribed by those who devised and trialled the test. However the distinction between tests and non-test assessment is not always all that clear. Some 'tests' can be absorbed into classroom work and look very much like normal classroom work as far as the children are concerned and so they cannot always be regarded as 'formal'. Whether formal or informal, of course, tests are only part of assessment.

Pupils are assessed for a number of reasons and the method chosen should suit the purpose. The main groups of purposes are:

☐ to help teaching (formative assessment)

To be effective in this role the assessment must be planned, recorded and used.

☐ to provide a record of achievement (summary)

This record is used in ongoing discussion with pupils, in producing a summary of progress and achievements for discussion with parents, and providing information to receiving teachers.

☐ as a contribution to school effectiveness measures

The information here will be about the performance of groups of pupils and should be only one part of the information about the school.

☐ to provide national performance survey data

This may be used to compare performance across various groups of pupils with national targets, to make comparisons from year to year and between sub-groups.

In the present context we will confine discussion to the first of these purposes since it is the one which has most impact on the teacher's day to day work.

Assessment as part of teaching

For pupils to have opportunity for learning with understanding they must have chance to apply the ideas and skills they bring with them to a new experience, and to take part in changing their ideas through using process skills (which are thereby developed). This is the essence of active learning by the child and involves mental and physical activity. The outcome will be ideas which make sense to the child and can be described as being 'owned' by the child.

For there to be the chance of this kind of learning the teacher must find out what are the ideas and skills that children bring to their learning experiences. This is the second step in teaching for understanding, the first being to provide the opportunity for children to engage with materials and problems in an informal way a which will allow their ideas to be elicited. These first and the subsequent steps are represented as follows (derived from the primary SPACE[1] project).

Providing opportunity for exploration and involvement

↓

Finding out ideas

↓

Interpreting

↓

Helping children to develop their ideas and process skills

↓

Assessing change in ideas and process skills

Where work continues on the same topic, the second and fifth steps become the same, both being concerned with ascertaining children's ideas at a particular time. The step of 'providing opportunity for exploration and involvement' ensures that children's thinking is engaged before ideas are sought. When introducing a new topic this initial step will merge into that of 'finding out ideas' where the teacher should be able to choose from a range of assessment methods, including open questioning, asking children to draw and annotate their drawings, encouraging children to write about or talk about their ideas, and at all times listening to what the children have to say.

The 'interpretation' is an important step which may be very short or quite long, depending on the teacher's experience and the nature of the ideas which come from the children. It involves reflecting on the experience which may have lead children to their current ideas and the kinds of reasoning that they are using. It is these things which are the basis for deciding the best way to help children in the next step. For example, if a child expresses the idea that growth only happens at certain times and is not continuous, the problem may be lack of evidence and the teacher will try to provide new experience which would challenge a view of discontinuous growth. On the other hand, if a child's ideas are highly bound to particular contexts (such as having different explanations for evaporation in different situations) it may not be new evidence which is required, but a discussion of the reasoning, or perhaps the words, being used about evidence already available.

For the step of 'helping children to develop their ideas' the starting point is the ideas which the children have, whether or not these ideas are along the 'right' lines. 'Development' means heightening, extending and strengthening useful ideas as well as challenging ones which are not useful in explaining things. One of the main ways of doing this involves children in devising and carrying out investigations to test their ideas, but some activities may be less extensive, such as asking children to provide examples of what they mean by certain words they use, encouraging them to apply ideas they use in one context in trying to explain another one, providing a greater range of evidence (including the use of secondary sources) and encouraging reflection and communication. During these activities there are likely to be opportunities for assessment of any change in children's ideas, using the same range of techniques as in the 'finding out' step.

The way in which the steps are described and characterised varies to some extent according to different interpretations of constructivism (see Chapter 3 page 25), but the important common feature is that assessment is embedded in the teaching. It is something that has to be given as much attention in planning as are the materials and the initial activities. Indeed the way in which the materials are used and the nature of the learning experiences will depend on the information gained in the initial assessment.

For example, a teacher planning a lesson on the energy value of different foods started by asking children to look at the information about

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energy value given on food wrapping and containers. When they began talking about food, the teacher found that many children did not connect food with energy at all. In fact it was sometimes the reverse; they reported feeling sleepy and not at all energetic after a big meal. Their ideas were simply that 'Food keeps us alive. You die if you don't eat.' The teacher then modified his plan and led a discussion on the different reasons the children could find for eating. These were gathered from the family at home, from claims made in advertisements, from the school cook. Then they discussed what happened if people did not eat, as well as the reasoning behind the views they had collected. After a few weeks the children had linked food and energy in their minds and so the teacher suggested the investigation of differences in this respect since this had become a real issue for the children.

Similarly a teacher finding children interpreting 1/2 x X as X divided by a half would recognise that the meaning of multiplication by a fraction needed to be established by practical examples before the rule for dividing by a fraction was introduced.

The particular problems of assessing active learning

It follows from the above discussion that for assessment to have a role in teaching it must comprehensively encompass all learning aims, whether of skill, concept or attitude development. This and other points made so far apply to all subjects, but there arise some particular problems shared by mathematics and science. Active learning, where mental and physical skills are being developed, and where *how* things are done is as important as *what* things are done, poses a challenge for assessment. Whilst much can be done through study of the products of children's work, it is also important to obtain information about, for example in a mathematics problem, how results were arrived at, whether the appropriate mathematical model was used, how operations were planned, what checks were carried out. In science the equivalent matters include whether tests carried out were 'fair', whether necessary variables were controlled, whether all evidence was taken into account and valid conclusions drawn from it.

Experienced teachers pick up this information during their normal interaction with children as part of teaching, but it poses considerable problems for novice teachers or for those who are introducing active approaches to learning in their classrooms for the first time. These teachers need help to use assessment in the pursuit of active learning, for which conventional methods of assessment are inadequate.

The help teachers need in relation to assessing active learning includes guidance in these three matters:

- what to assess
- how to collect information systematically
- how to involve children in the process.

Our discussion of these, whilst focusing on science and mathematics, has relevance to assessment in other parts of the curriculum.

What to assess

Assessing children during active learning is not just a matter of seeing whether or not they are observing, hypothesising, selecting appropriate models, checking, etc. Such broad judgements would have little value for helping the children's learning and in any case could not be made without first thinking through what it means to observe, hypothesise and so on. A first step, therefore, is to have some *general indicators* of what children are doing when carrying out the processes. The form these might take is best suggested through an example. At a workshop for science teacher educators [2], the following indicators were among those identified for process skills:

Observing

- using the senses (as many as safe and appropriate) to gather information
- identifying differences between similar objects or events
- identifying similarities between different objects or events
- noticing fine details that are relevant to an investigation
- recognising the order in which sequenced events take place
- looking for patterns that may exist in observations
- etc

Finding patterns and relationships

- putting various pieces of information (from direct observation or secondary sources) together and inferring something from them
- using patterns or relationships in information, measurements or observations to make predictions
- identifying trends or relationships in information
- realising the difference between a conclusion that fits all the evidence and an interference that goes beyond it
- etc

Hypothesising

- attempting to explain observations or relationships in terms of some principle or concept
- applying concepts or knowledge gained in one situation to help understanding, or to solve a problem in another
- recognising that there can be more than one possible explanation of an event
- realising the need to test explanations by gathering more evidence
- etc

Raising questions

- asking questions which lead to enquiry
- asking questions for information
- asking questions based on hypotheses
- realising that they can find out answers to some of their questions by their own investigation
- putting questions into a testable form
- recognising that some questions cannot be answered by enquiry
- etc

Devising investigations

- deciding what equipment, materials, etc are needed for an investigations
- identifying what is to change or be changed when different observations or measurements are made
- identifying what variables are to be kept the same for a fair test
- identifying what is to be measured or compared
- considering beforehand how the measurements, comparisons etc, are to be used to solve the problem
- deciding the order in which steps should be taken in the investigation
- etc

Attitude indicators can be defined similarly [3]:

Respect for evidence

- reporting what actually happens even if this is in conflict with expectations
- querying and checking parts of the evidence which do not fit into the pattern of other findings
- querying a conclusion or interpretation for which there is insufficient evidence
- treating ideas or conclusions as provisional and as being open to challenge by further evidence

Critical reflection

- willingness to review what they have done in order to consider how it might have been improved
- considering alternative procedures to those used
- identifying the points in favour and against the way in which an investigation was carried out or its results interpreted
- using critical reflection of a previous investigation in planning and carrying out a later one

A valuable workshop activity is for groups of teachers to work out these indicators for themselves. In doing so they will be clarifying the meaning of the process skills and attitudes and acquiring some ownership over the common definitions to be used. The indicators have to become part of the mental framework the teacher carries in his/her head, used in gathering information through watching children, listening to them, discussing with them what they are doing, as well as from any products in the form of writing, drawings or artefacts.

In use the general indicators have to be translated into the context of specific activities. What will children be doing or saying as evidence of the skills and attitudes when they are investigating vegetation along a transect, working out prime numbers, finding the relationship between masses on a balance - or investigating the melting of ice?

A class of 7 and 8 year olds was entranced by a very large block of ice (made by filling a balloon with water and putting it in freezer for a few days) floating in water [4]. In their interaction, discussion and related investigations they showed evidence of -

Observing

- they pointed out details of 'lines' and air bubbles in the ice and places where it was opaque
- they used their sense of touch to feel the 'stickiness' of the ice when just out of the freezer
- they noticed the sequence in which parts of the block started melting.

Finding patterns and relationships

- they linked together pieces of information in finding that the larger pieces which they broke off slid down a slope more easily than smaller pieces
- they found a pattern relating the size of pieces of ice to how quickly they melted
- they noted that the parts of the block in the water were melting first but showed caution in saying that 'it isn't everything that will melt more quickly in water than in air'.

Respect for evidence

- they reported evidence contrary to their ideas: 'moisture still forms on the outside of the tank when there is a cover on it, but I thought it wouldn't'
- querying whether there is air in the bubbles in the ice: 'we don't know the bubbles are air, we think they are'.

Critical reflection

- criticising their investigation of the effect of size on rate of melting: 'it would have been better to start with some larger pieces, then the difference would have shown up more'
- criticising a comparison of melting in and out of water: 'we should have held the piece in air above the table so that they weren't sitting in the water when they melted'.

Again, practice in 'translating' general indicators into evidence in the context of particular activities can be usefully carried out in teachers' workshops, where added value comes from considering the potential for learning in various activities. The importance of maintaining rigour in the assessment has to be emphasised; the indicators are the criteria against which the children's performance is assessed. Whilst the particular ways in which the skills and attitudes are made evident vary infinitely in various activities, it must be possible to show that they are all variants of the behaviours described by the general criteria.

Assessing systematically

Teachers new to the idea of assessing children during their activities can be overwhelmed by the scale of the task. How can all the children be observed all the time? They can't be, of course. What is required is for the teacher to plan to observe and make notes about one particular group during an investigation, which could spread over several sessions. The teacher would not be standing and watching this group for long periods; indeed the special focus in his/her mind should not be apparent to the children. The difference should be in the identification of particular kinds of information which the teacher is gathering during interaction with the group and in the notes (mental and perhaps written) made at the time about each child in the group. This is not in practice as demanding as it may at first seem, as the example of the ice block activities may have indicated.

In a subsequent investigation another group of pupils would be the 'targets' of observation and so on until all the members of the class have been assessed. A benefit of planning the assessment in this way is that information is gathered about all children, not just the ones who claim most of the teacher's attention. A consequence of this approach, however, is that pupils will be assessed on the same skills but when engaged upon different activities. The question as to whether this matters takes us back to the point about careful application of criteria. It is a useful focus for discussion in a workshop, as is the development of skill in assessing individual pupils working within a group. Ideally teachers or students should try out suggestions in classes in between workshop sessions.

Involving children in their own assessment

Involving children in assessment has several benefits. It can ease the teacher's burden of assessment, but perhaps more importantly enables the children to take a positive role in their learning. But it means that children must know what are the aims of their learning. Communicating these aims is not easy since directly telling about complex learning objectives and criteria of achievement is unlikely to be successful. So self-assessment skill has to be developed slowly and in an accepting and supportive atmosphere. It takes time to work through several stages before children are able to apply to their achievement anything like the criteria which their teacher would apply.

The process can begin usefully if children from about the age of eight are encouraged to select their 'best' work and to put this in a folder or bag. Part of the time for 'bagging' should be set aside for the teacher to talk to each child about why certain pieces of work were selected. The criteria which the children are using will become clear. Whatever they are, they should be accepted; they may have messages for the teacher. For example if work seems to be selected only on the basis of being 'tidy' and not in terms of content, then perhaps this aspect is being over-emphasised.

At first the discussion should only be to clarify the criteria the children use. 'Tell me what you particularly liked about this piece of work?' Gradually it will be possible to suggest criteria without dictating what the children should be selecting. Through such an approach as this children may begin to share the understanding of the objectives of their work and will be able to comment usefully on what they have learned. It then becomes easier to be explicit about further targets and for the children to recognise when they have achieved them. This is part of building confidence in pupils that their part in assessment is valued and that it can make their learning more enjoyable.

Using a somewhat similar approach to teachers in training has the same value for them as learners. Regularly they should be asked to comment on what part of their course they enjoy most – and least – and why. They should also be asked to identify what they have learned, as distinct from what they have done, and to reflect on the circumstances which affected their learning. They will then realise, from their own experience, the value of being asked to assess their own work and may then be more likely to give their pupils this opportunity.

w.h.

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6 The Training Needs of Teachers

Summary: The previous three chapters have been concerned with the dominant influences on teachers' decisions about children's learning experiences, consistent with the model presented in Chapter 1. This chapter now puts these factors into the context of the full range of skills and abilities which primary teachers need. A list of opportunities for developing these skills and abilities is proposed, in which a distinction is made between what is possible in pre-service courses and what may need to be provided by in-service. The implications of the trainee teachers' previous experience for the way in which the items on the list are provided is discussed briefly (a matter taken further in Chapter 7). Finally there are some points for consideration concerning the extent to which mathematics and science teacher education courses can be integrated.

Introduction

In Chapter 1 we presented a model of teaching and learning which, in a very telescoped way, indicated the decisions teachers have to make in providing learning experiences for their pupils. The particular point then made through this model was that these decisions were strongly related to a view of the subject, a view of learning and the evaluation of progress in learning. This indicated three priorities for teacher education courses: to ensure that teachers will receive a thorough understanding of how children learn, insight into the nature of scientific and mathematical activity and the ability to assess progress in all the objectives of learning and to use this information in providing learning experiences. These are such key points that we have devoted the three previous separate chapters to them.

Although these are essential, they are not, of course, sufficient as a preparation for teaching. Teachers also need knowledge of how to plan programmes, of what teaching and learning materials are available and how to choose and use them, of the pros and cons of different curriculum and class organisations, of the school and local authority organisation and the part they play in them. They need skills of managing their classroom and its resources, of responding to children's questions, of encouraging children, of intervening, of standing back, of assessing and keeping records, of matching demands to children's abilities to respond to them. They need attitudes of caring and responsibility, self-criticism, reflection, enthusiasm and optimism.

In this chapter we consider how to express these needs and propose a list to initiate discussion. It is neither a syllabus nor a set of objectives. The approach to identifying needs through specifying the skills, attitudes, knowledge and understanding required for teaching has certain attractions but also has dangers. Like the objectives approach to developing classroom programmes, the danger lies in the likelihood of identifying only what is easily definable, narrowing the range to what we are able to specify at present. In the case of classroom activities it is often found that we can with confidence recognise and specify worthwhile classroom experiences but yet not say precisely what learning we expect to arise from each experience. Similarly, it may be best to express the requirements for training as opportunities which have to be provided, which relate to the tasks teachers have to carry out, rather than to attempt to list specific learning outcomes. In any case, the training opportunities have to be identified for any action to be taken and so need to be identified at some stage, whether or not outcome objectives are also stated.

The balance between pre-service and in-service teacher education

Recognising that all the aims of training cannot be achieved in an initial course, it may be helpful to distinguish between what should be provided in pre-service courses and what could or should be provided through in-service courses. In attempting to draw this line there will inevitably be contention about what is essential as a 'basic minimum' for the beginning teacher and the matter will generally be decided in specific cases by the constraints of time and the consideration of the context of the training.

Time is necessarily limited in initial teacher education courses, especially where, as is usually the case in primary education, the teachers are being prepared to teach all subjects right across the curriculum. The timing and structure of pre-service courses also limits what can be achieved for the development of certain attributes of effective teaching requires experience of a sustained relationship with children and colleagues, and perhaps also with pupils' parents, which is generally not available at the initial stage. Therefore, where these attributes are not essential for the beginning teacher, their development need not be an aim of an initial course designed for the regular classroom teacher.

In this argument, however, there is an assumption about the provision of opportunities for continued professional development which should be available. The pace of change in teaching and the need to maintain the relevance of curriculum to children's everyday lives mean that in-service education is essential for other reasons than complementing pre-service education. However, if it is not available as an entitlement, then more than the base-line minimum has to be included in the initial course. What is possible in this respect varies with course structures. For example, courses where students spend a large portion of the time in schools will differ from those which are mainly institution based.

With these points in mind, then, we propose the following lists for discussion.

Proposals for core opportunities to be provided in initial teacher education courses:

- experience of the nature of scientific and mathematical activity
- experiences which generate an enthusiasm for science and mathematics
- activities which supplement personal knowledge and understanding of the subjects to a level beyond that expected of the children they are to teach
- experiences which inculcate the scientific attitudes of willingness to tolerate uncertainty, respect for evidence and open mindedness, and the professional attitudes of empathy, willingness to take responsibility and integrity
- activities which develop personal understanding of science processes and mathematical thinking
- studying the development of scientific and mathematical understanding and relating it to learning in general (ie developing a view of learning)
- studying and practising assessment of pupils during regular work and through structured tasks
- using the information so gained to match activities to pupils' progress
- planning the content and organisation of science and mathematics activities for children within a given school programme
- observing, evaluating and practising strategies for classroom control, with particular reference to practical science and mathematics activities
- studying a range of teaching styles and practising some of them
- acquiring familiarity with the required syllabus and with available published resources
- identifying and using criteria for selecting and adapting available classroom material
- experience of selecting and improvising simple equipment
- experiences designed to develop skill in handling and using productively children's questions
- studying and practising ways of encouraging children's communication and recording for various purposes and audiences
- making use of information technology in science and mathematics activities
- experience of techniques for, and encouragement of the habit of, self-appraisal

- study of where and how science and mathematics can usefully be combined in integrated or cross-curricular topics and approaches
- recognising and avoiding sources of bias or inequality of opportunity relating to gender, ethnicity, and physical handicap.

Additional opportunities which may be provided through in-service programmes

The needs of teachers for continued professional development and the extension of responsibility beyond their classroom requires opportunities for

- further development of all skills, attitudes, knowledge and understanding indicated in the above items
- studying ways of catering for children with special needs
- experiencing programme planning at the whole school level
- developing skills of working with colleagues and of liaison with other schools, both primary and secondary
- interpreting and using records and results of assessment
- reporting to parents and to school board members (if appropriate)
- communicating to parents and the local community the objectives of the school and dealing with queries
- selection and organisation of equipment and resources
- continued development of the use of information technology in teaching and learning science and mathematics
- developing classroom research skills
- incorporating science and mathematics experiences into the curriculum of pre-school children.

The learning of teacher trainees

In discussing children's learning in Chapter 3 we have made much of the importance of taking their earlier experiences and ideas derived from it as the starting point. This applies to all learning at all stages, to adults, to teacher trainees, as well as to children. Few of those entering teacher training courses will, in their own education, have had experiences which enable them to understand science and mathematics in the way we have suggested. Instead they will have notions of school and of these subjects, ingrained during up to twelve years of first hand experience, which conflict with the notions advocated here. It will, then, be as little effective just to tell them about a different view of education as it is just to tell children what are the 'right' ideas when their heads are full of their own ideas derived from their experience.

Trainee teachers' ideas of teaching are, therefore, likely to have to be changed, but changed in a way which gives them ownership of the new ideas. This means that the ideas make sense in terms of experience and reasoning, both of which have to be provided in the training course. Thus the course must give them the opportunities for realising the nature and the value of the kind of learning which leads to ownership through themselves learning in this way.

So, for example, when learning about the nature of scientific and mathematical activity, they should be working in the same way and it is intended that they work with children; being active physically and mentally. There should be some time in which they can undertake simple scientific investigations and mathematical tasks in a way which creates the excitement and enthusiasm of finding solutions through their own activity. After experiences of this kind, however, as adults and teachers, they can stand back and consider what they have been doing and examine their learning. They will then be able to think out for themselves how to make these sorts of experiences available to children. Then their previous assumptions about teaching will be challenged and their ideas changed.

This way of learning in teacher education does not have to be restricted to developing personal knowledge of science and mathematics and considering appropriate activities for children. It can and should be applied in all the experiences listed above. It means using the ideas which are already present in the trainees as the starting point, whether these are about how to answer children's questions, about how to assess children's understanding, about class organisation or about using equipment. In all these cases active learning can be implemented, producing an atmosphere where evidence and logical reasoning is used and everyone's ideas are respected and openly discussed.

A further advantage of this approach is that it fosters positive attitudes such as respect for evidence, open mindedness, tolerance and willingness to review actions and arguments critically. Recalling that attitudes are caught not taught, however, means that this has implications for the training of teacher educators.

The possibilities for some integration of mathematics and science in teacher education courses

The fact that a single list of opportunities covering both science and mathematics has been proposed here, suggests that there are many similarities in what is required by primary teachers in relation to these subjects. The possibility of time-conserving cooperation appears to be present, but nevertheless it would be perfectly logical to run two separate courses, with one providing the opportunities in relation to mathematics and the other having the same function for science. Course structure often determines the extent to which the teaching of science and mathematics can be studied together or in cross-curricular topics. Many courses begin from study of separately identified subjects so that these can be recognised in integrated topics which are introduced later; others begin in a more holistic way which requires considerable collaborative planning on the part of those involved in a primary teacher education course.

Since national or local curriculum guidelines usually exist as subject based documents, if integration is desired then it is important for teacher education to avoid further reinforcing the subject divisions through its course structures. This joint planning and shared sessions in many areas of professional development would not only assist in the 'litre into the half-litre pot' problem, but would be the best means of educating primary teachers – as opposed to teachers who change personae from 'science teacher' to 'maths teacher', from one part of the day to the next.

The way in which the teacher education course is organised may well have to strike a balance between what the logic of the two subjects dictates and what is expected in the schools (which does not always follow this logic). Teachers will need not only to consider but to experience some integrated work in their initial training if they are to practise it with understanding. It will only deepen their understanding of the subjects to consider the points of similarity and difference such as have been brought out in earlier chapters.

This matter is leading us from the subject of *what* opportunities are needed, which has been the concern of the present chapter, to the next chapter, in which the *how* is considered. In comparing the various approaches described in the next chapter, the sorts of experiences identified here have to be borne in mind. It is also necessary to remember that we wish to engender in teachers understanding and commitment, which must involve a process more appropriately described as 'education' rather than 'training'.

w.h.

7 Teacher Education Approaches

Summary: All that has been said in earlier chapters about the importance of learners' own activity has an impact on the methods used in teacher education courses. It is suggested that in all three dimensions of teacher education programmes – subject matter, pedagogical knowledge, classroom practice – the teaching approaches should foster trainees' activity and reflection. Differences between the demands of mathematics and science require corresponding differences in teacher workshops, whilst similarities argue for common elements. If differences and commonalities are to be balances, a 'mixed-economy' is suggested, in which differences are not neglected and the commonalities are exploited.

'Activity' as the key concept

The history of learning theory shows an increasing attention to learners' activities (see Chapter 3). Olsen, a Norwegian mathematics teacher educator, uses the word 'Activity' with a capital A to stress that real learning takes place when situation, problem and activities have a 'political' value for the students [1]. What they do must be of importance to their own lives, they must experience an improvement of circumstances because of their learning activity. The concept of 'ownership' (Chapter 6) has similar import. This is what student teachers should experience in their courses; activities which are important to their professional lives, as teachers-to-be now and eventually as teachers in their own classrooms.

Close to the concept of activity are 'interaction' [2], 'reflection' and 'production'. Working together in small cooperative groups stimulates those actions in which intuitive notions, beliefs, former experiences, preconceptions and informal procedures can be brought in, discussed and accommodated. Meanings are developed and shared as participants put their ideas into words to explain them to others. To do this, reflection (thinking about your own activity, even about your own thinking) appears to be necessary.

Teacher educators, who teach this view of learning in their courses, cannot neglect it in designing their own teacher education. This means that in the courses there must be room for cooperative work in which attention is paid to the students' subjective theories, their own learning-histories, their beliefs and conceptions with regard to the school-subjects they study, and their philosophy of the disciplines behind these subjects. It also means that teacher education courses must provide room for production, reflection and interaction in situations, and with problems to solve which future teachers recognise as real problems and which strongly motivate them. Sometimes conflicts arise, for instance, when personal theories do not fit into new ideas about learning and teaching. In this case the teacher educator is challenged to create a learning environment for students in which interaction, reflection and their own ideas set the course.

Three dimensions of activity in teacher education

Many primary teacher students have received from their own education a legacy of failure, or at least dissatisfaction, in relation to mathematics and science. Therefore, their first and foremost requirement is to acquire confidence, to gain an appreciation of the nature of scientific and mathematical activity, and to develop enthusiasm for teaching mathematics and science. The educator of teachers who is guided by this idea creates opportunities for participation and investigation by the students and thinks in terms of 'learning environments'.

Students must be active in the learning environment, in relation to the (school) subject matter, to the pedagogy of the subject and to what is going on in classrooms when this subject is being taught. Within this we can identify three dimensions of activity in teacher education:

(i) studying the (school) subject means that the student teacher's *own knowledge and ability* is brought up to the required level. If possible the studies extend around the subjects so that further relationships and deeper insight into the discipline emerge. Sometimes the history of science and mathematics can contribute, particularly when developments in the past give cues for learning in classrooms now. Mastery of the subject for teachers does not mean advanced study beyond the basics (see Chapter 4).

(ii) developmental work and research in both domains contributes to the body of *pedagogical knowledge*. This knowledge becomes the subject of activity in teacher education. There, as the (school) subject comes to life closer to classroom practice, students are further stimulated to improve their own standards of knowledge, but now from the perspective of being a teacher.

(iii) *classroom practice* is viewed as an experimental field in which learning and teaching can be investigated, designed and practised.

If we compare science and mathematics in the framework of these dimensions, similarities and differences emerge. For example, with respect to the first dimension, in primary mathematics the teacher's own skill in calculation plays a more important role than in science. It is that instrumental character of mathematics (Bishop speaks of 'symbolic technology'.[31]) that makes a difference in the pedagogical dimension as well. Student teachers often have to work hard to acquire the basic skills of mathematics, which they should have memorised during their own years of primary school, but often did not do. In science there is less need for memorised factual knowledge.

In both subjects a large amount of psychological research has taken place from which many contributions to pedagogical knowledge have come. Together with the results of developmental projects, this means that a considerable amount of knowledge is available with regard to both subjects.

Science and mathematics are also very close in the third dimension. Learning to teach in the classrooms requires a study of learning processes, knowing how to start those processes and how to support and supervise them and what materials can be used.

Activity in teaching workshops

A workshop is a direct way of providing a learning experience in which the learner creates meaning or understanding through his or her mental or physical activity. What is provided as a basis for this action can be objects or materials to investigate or use, or problems to solve, or evidence to examine and discuss, or all these together. The outcome may be an artefact, a solution to a problem, a plan, the recognition of a new relationship between things, a critique or a set of criteria. Perhaps the most important product, however, is a greater understanding of how to achieve such results.

Workshops of this kind have been well developed in science[4]. Not only is the scientific phenomenon at hand the subject of investigation, but so are the investigators and the investigation itself. All those involved stand back from their involvement and reflect on their role, their interaction with each other and with the materials and consider the role of the materials used and the educational setting. These workshops in science have the three characteristics of the learning environment for student teachers, mentioned earlier. They can also provide opportunities for the activities listed on pages 55 and56.

This kind of workshop is in many respects quite similar to what in mathematics is called a 'mathematical-didactical' workshop. Student teachers cooperatively and thus interactively solve mathematical problems at their own level, similar to the way children would do it at their levels. All the three dimensions are taken care of, so that not only the mathematical subject matter, but the pedagogical knowledge and the learning and teaching of this subject matter become issues of study as well. A few examples illustrate specific workshops, for

- mathematical problem solving
- explaining mathematics
- analysing concrete materials for use in primary schools
- educational design
- creating help for low achievers.

But learning to teach mathematics requires more than just the mathematics to be considered [5]. The educator of primary mathematics teachers must pay particular attention to issues like:

- studying children's strategies in acquiring number concepts and the way school books teach it
- observing children's learning of algorithms and the way school books teach it.

Investigating the differences

'Learning to teach science' and 'learning to teach mathematics' are to be distinguished. There are essential differences in the nature of knowledge in the subjects. The following will illustrate this point of view.

Take the activity of *'organising'*, which in both, science as well as in mathematics, is considered as basic and fundamental. But, as we shall see, it has in each subject a very different interpretation and application. In the science workshop organising (the investigation) means to gather data systematically in order to get knowledge about the object of the current investigation, to be able to make a clear description of the phenomenon, to create possibilities for investigating details, to ask better questions, to manage the answers in order to check earlier hypotheses etc. Organising in science workshops serves the investigation.

Organising in mathematics not only facilitates the problem solving activity, but it also affects the way in which the knowledge is organised in children's minds. A specific organisation of 'situations of division' (see Chapter 2, p19) leads to a specific algorithm constructed in the children's minds. Using a city plan to organise multiplication-situations affects the way a related thinking model (in this case the grid model) arises in the minds of the pupils. This means that in mathematics organising *structures the knowledge* itself and, because such knowledge could be strongly context-bound, it is necessary to investigate other contexts with a similar structure (isomorphic problems) that permit the same organisation. From a broader point of view all kinds of organisation can be seen as part of 'mathematising'. In all particular situations mathematising stands for organisation, discovering a structure, creating a useful notation schema, inventing short-cuts. All these activities affect the structure of developing personal knowledge. (An example of organising in mathematics, is given as an Annex to this Chapter.)

The commonalities

In the foregoing paragraphs a number of similarities between primary science and mathematics passed in review. Summarising we conclude that learning to teach both subjects in primary school

- demands opportunities for student teachers to overcome dissatisfaction from the past and to create new perspectives on the subjects
- needs activity with interactive, reflective and productive participation in investigation
- has to take into account students' own ideas and personal, subjective theories
- must pay much attention to children's learning processes in science and mathematics
- is organised in a 'reflective model' of teacher education [6].

A 'mixed economy'

Balancing the differences and similarities it becomes possible to draw some conclusions. When designing teacher education it must be possible to realise collaboration between science and mathematics, but recommending total integration of both domains in teacher education, would be unwise. The identified differences ask for a clear distinction, the existence of fundamental commonalities suggest a carefully thought-out 'mixed economy', certainly not only to save learning time, but because a mixed economy of science and mathematics teaching in the colleges of education really can be richer than teaching the subject areas separately.

A mixed economy in teacher education for instance means combined science and mathematical pedagogic workshops, with maths activities in a science context, as starting points for a study of learning and design practice.

This mixed economy may be the best from the teachers' point of view. If the differences are clear and the teacher has enough self-confidence to teach both subjects 'actively', it will be worthwhile sometimes to use mathematics in order to deepen science understanding or to use a science context to develop a mathematical skill. Between the separated approach and the advanced integration of the two, many variants are possible. At the secondary school level one subject is used as a field of practice for the other. At the primary level the similarities in pedagogy weighs more heavily than the need to reflect the distinctions between the disciplines. These differences should be considered by both teacher educators and trainee teachers in relation to their own teaching.

The role of teacher education is to provide the knowledge and awareness which will support a greater proportion of integrated work. If students and teachers encounter mathematics and science only in separate compartments, their ability to combine the two is unlikely to grow. Some of their work should be clearly focused on each subject so that the identity of mathematical and scientific activity can be in no doubt, but some should be integrated so that points of contact can increasingly be recognised.

f.g.

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Annex

The six-column Erathostenes sieve.

The idea of building a sieve in order to identify all the prime numbers up to a certain pre-determined limit (eg up to 90) is a rather natural one: it uses only the concept of prime number per se and the idea of]getting rid' of all those numbers that are multiples of smaller ones. So accomplishment of the task can be organised as follows:

1. first write down all the natural numbers up to 90
2. eliminate 1 (which for well-known (?) reasons is not to be counted among the primes)
3. add the smallest non-eliminated number to the list of primes
4. eliminate all multiples of this same number
5. repeat steps 3 and 4 as often as necessary (what is the exact stopping rule?).

When asked to accomplish such a task, many people might have the tendency to display the original list of natural numbers in ten columns (at least if working in base ten):

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	...						
						...	88	89	90

But there are clearly no deep reasons for such a disposition of the numbers. Any more or less systematic display would also work. What about the following one:

2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31
32	33	34	35	36	37
38	39	40	41	42	43
44	45	46	47	48	49
50	51	52	53	54	55
56	57	58	59	60	61
62	63	64	65	66	67
68	69	70	71	72	73
74	75	76	77	78	79
80	81	82	83	84	85
86	87	88	89	90	

Search for the multiples of 2 eliminates the first, the third and the fifth columns; also search for the multiples of 3 eliminates the second column (the fifth having already been eliminated). So all other primes to be found will be either in column four or in column six. The following result is thus transparent, just from the way we have displayed the original list of numbers: *Except for 2 and 3, all primes are of the form* $6k \pm 1$.

It is worth noting also that elimination of multiples of other primes is greatly facilitated by the particular form of these primes. For instance, since $11 = 6 \times 2 - 1$, all its multiples are to be found on slant lines obtained by going down two rows and going left one column.

It is clear that the very idea of displaying the natural numbers on six columns **comes from** the prior knowledge of the result that primes are of the form $6k \pm 1$ (except for 2 and 3). So the original idea is motivated by some 'existing' knowledge. So in presenting such a display to, say, a student, it would be important to allow her/him to discover the result for her/himself: this illustrates clearly how crucial the organisation of information can be.

b.h.

Suggestions for further discussion, for research and for development

In the writing of these chapters and in the surrounding discussions the authors were aware of the many questions which the subject raises for policy makers, teacher educators and teachers in training. We see our writing as only the start in addressing these questions and an important outcome of our work being in the further discussion, research and development which may follow. We list here the points for discussion and the suggestions for research and development identified in the preparation of the final draft of this publication.

Questions for discussion

1. What is the basis of support for the proposition that the learning of mathematics and science has a common root and procedure?

2. Can a common approach to learning be supported at the same time as recognising the ways in which the subjects differ, for example in respect of evidence from the real world being the ultimate source of authority in science whilst the logic of reasoning having this role in mathematics?

3. Is this same constructivist approach to learning appropriate to other subjects?

4. What accommodations may need to be made where cultural expectations of children's behaviour conflict with the tenets of constructivist learning; where children's views and questions are not valued, for example?

5. To what extent is a constructivist approach to learning compatible with a detailed and highly structured curriculum or syllabus?

6. What are the limiting factors affecting the validity of the proposed model for teacher decision making? In particular how does the model apply where the teaching conditions are not of the teacher's choosing, for example in the matters of class size, availability of materials, a restricting curriculum?

7. To what extent is the rationale for the proposed model valid in different cultures and contexts?

8. What are the implications of the model and the approach to learning for the teaching of subject matter of mathematics and science in teacher education courses?

9. What can be done to help teachers in training resolve conflicting philosophies of education which may be implicit or explicit in different parts of their training courses?

10. To what extent can the workshop approach be applied in all aspects of teacher education?

11. What can be done to break into the vicious circle of society's generally negative attitudes to science and mathematics, which creates teachers with little confidence in these subjects who perpetuate the negative views in their pupils?

12. Does popularisation of science and mathematics help the primary teacher? If so, what further efforts can be made towards this end?

13.. Can mathematics 'fairs' be devised which serve the same purpose as school science fairs?

14. In the implementation of a unified approach to mathematics and science what steps need to be taken to avoid mathematics being treated as a toolkit for science?

15. What changes are needed in the assessment of children's learning in mathematics and science so that credit can be given for responses which may not be exact but which represent valuable steps in development of ideas and skills?

16. How can the encouragement of collaborative work and cooperative learning in the classroom be reconciled with the popular demand to assess pupils and students individually?

17. What are the implications for school and class organisation of approaches to learning which encourage pupils to express, use and discuss their own ideas?

18. In the context of problem solving and activity in mathematics and science how is a teacher to deal with children's questions answerable only by reference to complex concepts which are not intellectually accessible to primary pupils?

19. How are different degrees of 'concreteness' and 'abstractness' in mathematics and science topics to be identified and communicated to teachers?

20. How can the teacher's role be presented as going beyond the

provision of materials and including crucially the stimulation of pupils' reflection on their activity without seeming to support overly directive teaching?

21. What role is there in primary schools for specialists in primary mathematics and in primary science?

22. What are the implications for the secondary school curriculum of the need for primary teachers with a good background in science and mathematics?

23. How can students with a good interest and ability in science and mathematics be attracted to primary school teaching?

Proposals for research

i) An international survey along IEA lines of primary teacher education course content and methods.

ii) The documentation by teachers of their experiencesof attempting to apply a constructivst approach in order to feed back into teacher education information about the range of problems likely to be encountered.

iii) A long-term follow up study of a cohort of newly qualified teachers with a view to relating training experiences to later experiences in teaching.

Areas for development

i) Materials to support workshop activities in teacher education which are designed to convey the commonalities of science and mathematics education at the primary level.

ii) Case studies to exemplify teacher education courses where there are joint science and mathematics components.

iii) Classroom materials for teachers to use in a unified approach to learning mathematics and science such that the essential identity of the subjects are respected.

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